MA 265 Lecture 40

Section 8.5 Dynamic System

Consider a 2×2 homogeneous system of differential equations

$$\begin{aligned} \frac{dx}{dt} &= ax + by \\ \frac{dy}{dt} &= cx + dy \end{aligned}$$

If the independent variable t denotes time, then the above system is a **dynamic system**.

- The solution $x = x(t)$ and $y = y(t)$ represents
- The xy-plane is called
- A sketch of trajectory in the phase plane is called a
- $\frac{dx}{dt}$, $\frac{dy}{dt}$ represent velocity in x and y direction. A point in the phase plane at which both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are zero is called

Different Types of Equilibrium Points:

Stable **Unstable** **Saddle Point**

The coefficient matrix A will determine features of the phase portraits of the system.

Case 1: $\lambda_1 < 0$**,** $\lambda_2 < 0$

Case 2: $\lambda_1 > 0$**,** $\lambda_2 > 0$

Case 3: $\lambda_1 < 0$**,** $\lambda_2 > 0$

Case 4: $\lambda_{1,}$**,** λ_2 **are Complex Numbers**

Example 1. *Determine the phase portrait of the dynamic system:*

$$\mathbf{x}'(t) = \begin{bmatrix} -2 & -2 \\ 1 & -5 \end{bmatrix} \mathbf{x}(t)$$

Example 2. *Determine the phase portrait of the dynamic system:*

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & -1 \\ -2 & 0 \end{bmatrix} \mathbf{x}(t)$$