MA 265 Lecture 5

Section 2.1   Echelon Form of a Matrix

Definitions

An $m \times n$ matrix A is said to be in ______________________________ if

(a)

(b)

(c)

(d)

An $m \times n$ matrix satisfying properties **(a)**, **(b)**, **(c)** is said to be in ____________________.

We can define ____________________ and _________________________ in a similar manner.

Example 1. *Determine whether the following matrices are in (reduced) row echelon form*

$$A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 2 & -2 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

An **elementary row operation** on a matrix A is any one of the following operations:

(a)

notation:

(b)

notation:

(c)

notation:

Example 2. *Let*

$$A = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 2 & 3 & 0 & -2 \\ 3 & 3 & 6 & -9 \end{bmatrix},$$

Then

$$B = A_{r_1 \leftrightarrow r_3} =$$

$$C = A_{\frac{1}{3}r_3 \to r_3} =$$

$$D = A_{-2r_2 + r_3 \to r_3} =$$

An $m \times n$ matrix B is said to be ______________________ to an $m \times n$ matrix A if

> **Theorem.** Every nonzero $m \times n$ matrix $A = [a_{ij}]$

Example 3. *Find a matrix in row echelon form that is row equivalent to the matrix*

$$A = \begin{bmatrix} 0 & 2 & 3 & -4 & 1 \\ 0 & 0 & 2 & 3 & 4 \\ 2 & 2 & -5 & 2 & 4 \\ 2 & 0 & -6 & 9 & 7 \end{bmatrix}$$

Theorem. Every nonzero $m \times n$ matrix $A = [a_{ij}]$

Remark:

Example 4. *Find the reduced row echelon form of the matrix*

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 9 \\ 3 & 2 & -4 \end{bmatrix}$$