MA 265 Lecture 6

Section 2.2 Solving Linear Systems

Example 1. *Solve the linear system whose augmented matrix has the echelon form:*

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

To solve a linear system $A\mathbf{x} = \mathbf{b}$,

1. form the augmented matrix
2. transform to a matrix $[C \mid \mathbf{d}]$ in
3. find solution of the linear system w.r.t. $[C \mid \mathbf{d}]$ using

Remark:

If A is $n \times n$, and the linear system $A\mathbf{x} = \mathbf{b}$ has a unique solution, then

Example 2. *Solve the linear system*

$$\begin{aligned} x + 2y + 3z &= 9 \\ 2x - y + z &= 8 \\ 3x - z &= 3 \end{aligned}$$

In general, A is $m \times n$ and the augmented matrix $[C \mid \mathbf{d}]$ might have the following form

$$\left[\begin{array}{ccccc|c} 1 & c_{12} & \cdots & & c_{1n} & d_1 \\ 0 & 1 & c_{23} & \cdots & c_{2n} & d_2 \\ \vdots & \vdots & \ddots & \ddots & & \\ 0 & 0 & & 1 & c_{k-1,n} & d_{k-1} \\ 0 & 0 & \cdots & 0 & c_{k,n} & d_k \\ 0 & 0 & \cdots & & 0 & d_{k+1} \\ \vdots & \vdots & & & \vdots & \\ 0 & 0 & \cdots & & 0 & d_m \end{array}\right]$$

- If $d_{k+1} = 1$,
- If $d_{k+1} = 0$, $c_{k,n} = 1$
- If $c_{k,n} = 0$

Example 3. *Let*

$$[C \mid \mathbf{d}] = \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & -1 & 7 \\ 0 & 0 & 1 & 2 & 3 & 7 \\ 0 & 0 & 0 & 1 & 2 & 9 \end{array}\right].$$

Example 4. *Let*

$$[C \mid \mathbf{d}] = \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 6 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right].$$

Example 5. *Solve the linear system*

$$\begin{aligned} x + 2y + 3z &= 6 \\ 2x - 3y + 2z &= 14 \\ 3x + y - z &= -2 \end{aligned}$$

Homogeneous System

Example 6. *Solve the linear system*

$$\begin{aligned} x + y + z + w &= 0 \\ x + w &= 0 \\ x + 2y + z &= 0 \end{aligned}$$

Remark:

Relationship between nonhomogeneous and homogeneous systems

Let $A\mathbf{x} = \mathbf{b}$, $\mathbf{b} \neq \mathbf{0}$ be a consistent system. The associated homogeneous system is

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