MA 265 Lecture 7

Section 2.3 Elementary Matrices; Finding A^{-1}

Definition

An <u>$n \times n$ **elementary matrix of type I, II, or III**</u> is

Example 1. *The following are elementary matrices*

$$E_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Theorem Let A be an $m \times n$ matrix, and let an elementary row operation be performed on A to yield matrix B, then

Remark: Let A and B be matrices.

1. A is row equivalent to B if and only if

2. An elementary matrix E is

3. A is nonsingular if and only if

4. A is nonsingular if and only if

5. Let A be an $n \times n$ matrix. The homogeneous system $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution if and only if

Finding A^{-1}

A nonsingular matrix A is row equivalent to I_n, so that

$$A =$$

Hence

$$A^{-1} =$$

Performing elementary row operations to the matrix $[A \mid I_n]$, we obtain

Example 2. *Find A^{-1} if*

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$$

Theorem An $n \times n$ matrix A is singular if and only if

Remark

Example 3. *Find* A^{-1} *if*

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$$

Theorem If A and B are $n \times n$ matrices such that $AB = I_n$, then

Remark

Exercise Find all values of a for which the inverse of

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & a & 2 \end{bmatrix}$$

exists. What is A^{-1}?