MA 265 Lecture 8

Section 3.1 Definition of Determinants

Definition of Permutation

Let $S = \{1, 2, \cdots, n\}$ be the set of integers from 1 to n, arranged in ascending order.

We can consider a permutation of S to be a one-to-one mapping of S to itself. For example, let $S = \{1, 2, 3, 4\}$

Question: How many permutations does $S = \{1, 2, \cdots, n\}$ have?

Example 1. *Let $S = \{1, 2, 3\}$. Find all permutations of S.*

A permutation $j_1 j_2 \cdots j_n$ is said to have an **inversion** if

A permutation is called **even** if

A permutation is called **odd** if

Example 2. .

- S_1

- S_2

- *The permutation* 4312 *is in* S_4

- S_3

Definition of Determinant

Let $A = [a_{ij}]$ be an $n \times n$ matrix. The **determinant** function,

Remark

Example 3. *Calculate* $\det(A)$ *if*

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Example 4. *Calculate* $\det(A)$ *if*

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Another way to calculate $det(A)$, <u>if A is a 3×3 matrix</u>:

Example 5. *Compute the determinant*

$$\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix}\right) =$$