

MA 265 Lecture 9

Section 3.2 Properties of Determinants

Properties of determinants

Let A and B be $n \times n$ matrices. Their determinants have following properties

1. $\det(A^T) =$
2. If $B = A_{r_i \leftrightarrow r_j}$ or $B = A_{c_i \leftrightarrow c_j}$ then
3. If two rows (columns) of A are equal, then
4. If a row (column) of A consists entirely of zeros, then
5. If $B = A_{kr_i \rightarrow r_i}$ or $B = A_{kc_i \rightarrow c_i}$, then
6. If $B = A_{kr_i + r_j \rightarrow r_j}$ or $B = A_{kc_i + c_j \rightarrow c_j}$, then
7. If $A = [a_{ij}]$ is upper (lower) triangular, then

Example 1. Find $\det(A)$ if

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 4 & -2 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Properties of determinants (contd)

8. If E is an elementary matrix, then

9. If A is an $n \times n$ matrix, then A is nonsingular if and only if

10. If A and B are $n \times n$ matrices, then

Example 2. *Verify the property 10 using the matrices*

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

Example 3.

- *If A is nonsingular. $\det(A^{-1}) =$*

- *If A and B are $n \times n$ matrices, is $\det(AB) = \det(BA)$?*