# MA 266 Lecture 1

## Section 1.1 Mathematical Models; Direction Fields

Question: What is a differential equation?

A differential equation is equation containing derivatives.

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Example 1. ***(Types of equations)***

1. *Find $x$ in* $x^2 + 2x + 1 = 0.$ ← quadratic equation

2. *Find $f(t)$ in* $f(t)\cos(t) = e^t - \sin(t).$ single variable equation

3. *Find $y(t)$ in* $y'' + 3y' = e^t.$ ← differential equation

Question: Why do we study differential equations?

- Many principles or laws in physics are relations involving differential equations.
- In mathematical terms, relations are equations, and rates are derivatives. Equations containing derivatives are differential equations
- A differential equation that describes certain physical process is often called a mathematical model

Example 2. *(An example of mathematical model — A falling object)*

*Consider an object with a mass $m$ falling near the sea level. Formulate a differential equation to model its motion.*

- **Notations** $m$ – mass of the object

  $a$ – the acceleration

  $F$ – net force exerted on the object

- **Physical Law:** Newton's second law

$$F = ma = m\frac{dv}{dt}$$

- **Forces that acted on the object**

gravity – $m \cdot g$

drag force – $\gamma v$, $\gamma$ – drag coefficient kg/s

$$F = mg - \gamma v$$

$$\Rightarrow \quad m\frac{dv}{dt} = mg - \gamma v$$

**Remark** The falling object model contains three constants: $m$, $g$, and $\gamma$

- $m$ – mass
- $g$ – gravity constant

$\gamma$ – drag coefficient

## Direction Fields

We let $m = 10kg$ and $\gamma = 2kg/s$ in the falling object model, so it becomes

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}.$$

**Basic idea of direction fields:**

direction field can be constructed by evaluating $f$ at each point of a rectangular grid. At each point, a short line segment is drawn whose slope is the value $f$ at ~~that~~ that point.

**How to construct Direction Fields?**

If we let $v = 40$, then

$$\frac{dv}{dt} = 1.8$$

If we let $v = 50$, then

$$\frac{dv}{dt} = -0.2$$

Note that if $9.8 - \frac{v}{5} = 0$, then

$v = 5 \times 9.8 = 49\ m/s$,

which is the equilibrium solution or terminal velocity.

**Remarks on Direction Fields**

Direction fields are valuable tools in studying differential equations of the form

$$\frac{dy}{dt} = f(t, y) \leftarrow \text{rate function}$$

Two things about direction fields

- we evaluate $f(t, y)$ many times.
- we can use computer to draw direction fields.

**A MATLAB Implementation on Direction Fields**

1. Download the MATLAB file **dfield8.m** from

   `http://math.rice.edu/~dfield`

2. Type **dfield8**, at MATLAB command window.
3. In the popup window, enter your differential equations, and the range of independent and dependent variables.
4. Hit **Proceed** to see the direction field of your differential equation.

**Example 3.** *Draw a direction field of the each of the following differential equations, then determine the behavior of the solution as $t \to \infty$.*

$$(1)\ y' = 3 - 2y, \qquad (2)\ y' = 3 + 2y, \qquad (3)\ y' = -y(5-y).$$