# MA 266 Lecture 1

#### Mathematical Models; Direction Fields Section 1.1

Question: What is a differential equation?

A differential equation is equation containing derivatives

Example 1. (Types of equations)

- 1. Find x in  $x^2 + 2x + 1 = 0$ .  $\leftarrow$  quadratic equation
- 2. Find f(t) in  $f(t)\cos(t)=e^t-\sin(t)$ . Gingle variable equation
- 3. Find y(t) in  $y'' + 3y' = e^t$ .  $\angle$  differential equation

Question: Why do we study differential equations?

- Many principles or laws in physics are relations involving differential liquations.

   In mathematical terms, relations are against ons and rates are derivatives

  Equations containing derivatives are differential equations.
- A differential equation that describes certain physical process is often called a \_ mathematical mode

## Example 2. (An example of mathematical model — A falling object)

Consider an object with a mass m falling near the sea level. Formulate a differential equation to model its motion.

- m marss of the object Notations a - the acceleration F - net force exerted on the object
- Physical Law: Newton's second law

• Forces that acted on the object

**Remark** The falling object model contains three constants: m, g, and  $\gamma$ 

- m marss
- g gravity constant re 1 page 2 of 5

### Direction Fields

We let m = 10kg and  $\gamma = 2kg/s$  in the falling object model, so it becomes

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}.$$

Basic idea of direction fields:

Point of a vector and each point, a short line segment is alram whose If we let v = 40, then

Of the each point, a short line segment is alram whose v = 40, then

Of the each point, a short line segment is alram whose v = 40, then

If we let v = 50, then

Note that if  $9.8 - \frac{v}{5} = 0$ , then

which is the equilibrium solution or terminal velocity.

#### Remarks on Direction Fields

Direction fields are valuable tools in studying differential equations of the form

Two things about direction fields

- · we evaluate fit, y, many times.
- · we can use computer to draw direction filelds.

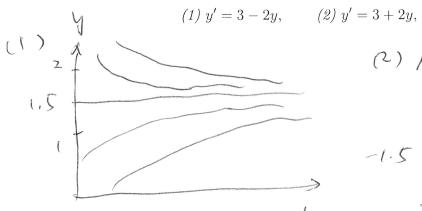
### A MATLAB Implementation on Direction Fields

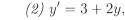
1. Download the MATLAB file dfield8.m from

http://math.rice.edu/~dfield

- 2. Type dfield8, at MATLAB command window.
- 3. In the popup window, enter your differential equations, and the range of independent and dependent variables.
- 4. Hit **Proceed** to see the direction field of your differential equation.

Example 3. Draw a direction field of the each of the following differential equations, then determine the behavior of the solution as  $t \to \infty$ .





(3) 
$$y' = -y(5-y)$$
.





