MA 266 Lecture 4

Section 2.2 Separable Equations

In this section, we use x to replace t as the independent variable.

The general form of a *nonlinear* first order equation is

It can be written in the form $M(x,y) + N(x,y)\frac{dy}{dx} = 0$

If M is a function of x only, and N is a function of y only, then the equation becomes

Such an equation is called separable eqn. A separable equation can be solved by

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Example 1. *Show that the equation*

$$\frac{dy}{dx} = \frac{x^2}{1-y^2}$$

is separable, and then find its solution (in implicit form).

$$-x^2 + (1-y^2)\frac{dy}{dx} = 0$$

$$\frac{df(y)}{dx} = \frac{df}{dy}\frac{dy}{dx} = f'(y)\frac{dy}{dx}$$

$$\frac{d}{dx}\left(-\frac{x^3}{3}\right) + \frac{d}{dx}\left(y - \frac{y^3}{3}\right) = 0$$

$$\frac{d}{dx}\left(-\frac{x^3}{3} + y - \frac{y^3}{3}\right) = 0$$

$$-x^3 + 3y - y^3 = c$$

In general, for any separable equation $M(x) + N(y)\frac{dy}{dx} = 0$, we let

$H_1' = M$, $H_2' = N$

$$H_1' + H_2' \frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{d}{dx}(H_1 + H_2) = 0$$

$$\boxed{H_1 + H_2 = C}$$

$$H_2' \frac{dy}{dx} = \frac{d}{dy} H_2 \frac{dy}{dx} = \frac{d}{dx} H_2$$

Example 2. *Solve the initial value problem*

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, \quad y(0) = -1,$$

and determine the interval in which the solution exists.

$$2(y-1)\,dy = (3x^2 + 4x + 2)\,dx$$

$$y^2 - 2y = x^3 + 2x^2 + 2x + c$$

$$y(0) = -1 \Rightarrow \quad c = 3$$

$$y = 1 \pm \sqrt{x^3 + 2x^2 + 2x + 4}$$

Example 3. *Solve the initial value problem*

$$y' = \frac{1+3x^2}{3y^2-6y}, \qquad y(0)=1.$$

and determine the interval in which the solution is valid.

$$\int 3y^2-6y\,dy = \int 1+3x^2\,dx$$

$$y^3-3y^2 = x+x^3+c$$

$$y(0)=1 \Rightarrow c=-2$$

$$y^3-3y^2-x-x^3+2=0$$

$$3y^2-6y=0 \Rightarrow 3y(y-2)=0$$

$$y=0 \text{ or } y=2$$

where the integral curve has a vertical tangent