

Formula sheet

Fourier series: For a $2L$ -periodic function $f(x)$, the Fourier series for f is

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L},$$

where for $n = 1, 2, \dots$,

$$a_0 = \frac{1}{L} \int_{-L}^L f(x) dx, \quad a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, \quad b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx.$$

Heat equation 1: The solution of the heat equation $\alpha^2 u_{xx} = u_t$, $0 < x < L$, $t > 0$, satisfying the (fixed temperature) homogeneous boundary conditions $u(0, t) = u(L, t) = 0$ for $t > 0$ with initial temperature $u(x, 0) = f(x)$ has the general form

$$u(x, t) = \sum_{n=1}^{\infty} c_n e^{-n^2 \pi^2 \alpha^2 t / L^2} \sin \frac{n\pi x}{L}, \quad \text{where} \quad c_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx.$$

Heat equation 2: The solution of the heat equation $\alpha^2 u_{xx} = u_t$, $0 < x < L$, $t > 0$, satisfying the insulated boundary conditions $u_x(0, t) = u_x(L, t) = 0$ for $t > 0$ with initial temperature $u(x, 0) = f(x)$ has the general form

$$u(x, t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n e^{-n^2 \pi^2 \alpha^2 t / L^2} \cos \frac{n\pi x}{L}, \quad \text{where} \quad c_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx.$$

Wave equation: The solution of the wave equation $\alpha^2 u_{xx} = u_{tt}$, $0 < x < L$, $t > 0$, satisfying the homogeneous boundary conditions $u(0, t) = u(L, t) = 0$ for $t > 0$ and initial conditions $u(x, 0) = f(x)$ and $u_t(x, 0) = g(x)$ for $0 \leq x \leq L$ has the general form

$$u(x, t) = \sum_{n=1}^{\infty} \sin \frac{n\pi x}{L} \left(c_n \cos \frac{n\pi \alpha t}{L} + k_n \sin \frac{n\pi \alpha t}{L} \right)$$

where

$$c_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx \quad \text{and} \quad k_n = \frac{2}{n\pi \alpha} \int_0^L g(x) \sin \frac{n\pi x}{L} dx.$$

Laplace equation: The solution of the Laplace equation $u_{xx} + u_{yy} = 0$, $0 < x < a$, $0 \leq y \leq b$, satisfying the boundary conditions $u(x, 0) = u(x, b) = 0$ for $0 < x < a$ and $u(0, y) = 0$ and $u(a, y) = f(y)$ for $0 \leq y \leq b$ has the general form

$$u(x, y) = \sum_{n=1}^{\infty} c_n \sinh \frac{n\pi x}{b} \sin \frac{n\pi y}{b} \quad \text{where} \quad c_n = \frac{2}{b \sinh(\frac{n\pi a}{b})} \int_0^b f(y) \sin \frac{n\pi y}{b} dy.$$

Figure 1: Laplace Transform Table

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	e^{at}	$\frac{1}{s-a}$
3.	t^n	$\frac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\frac{a}{s^2 + a^2}$
6.	$\cos at$	$\frac{s}{s^2 + a^2}$
7.	$\sinh at$	$\frac{a}{s^2 - a^2}$
8.	$\cosh at$	$\frac{s}{s^2 - a^2}$
9.	$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}$
10.	$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}$
11.	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\frac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right) \ c > 0$
16.	$\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$