QUIZ
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Quiz 10

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Integration by Partial Fractions

Evaluate the integrals:

(8 points) (a) $\int \frac{1}{x^3+x} dx$

solution:

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

Multiplying $x(x^2+1)$ on both sides, we obtain

$$1 = A(x^2+1) + (Bx+C)x = (A+B)x^2 + Cx + A$$

Let $x = 0$, we have $A = 1$. And we also have

$$\begin{cases} A + B = 0 \\ C = 0 \end{cases}$$

Thus, we have $A = 1$, $B = -1$, and $C = 0$.

$$\begin{aligned} \int \frac{1}{x(x^2+1)} dx &= \int \frac{1}{x} dx + \int \frac{-x}{x^2+1} dx \\ &= \ln|x| - \frac{1}{2}\ln|x^2+1| + C \end{aligned}$$

(12 points) (b) $\int \frac{e^x}{1-e^{2x}}dx$

[Hint: First use a substitution and then use partial fractions]

solution: Follow the hint, let $u = e^x$, then $du = e^x dx$.

$$I = -\int \frac{1}{u^2-1}du$$

$$\frac{1}{u^2-1} = \frac{1}{(u+1)(u-1)} = \frac{A}{u+1} + \frac{B}{u-1}$$

Multiplying (u^2-1) on both sides, we obtain

$$1 = A(u-1) + B(u+1)$$

Let $u = 1$, then $B = \frac{1}{2}$. Let $u = -1$, then $A = -\frac{1}{2}$. Thus,

$$\begin{aligned} I &= -\int \frac{1}{u^2-1}du = \int \frac{\frac{1}{2}}{(u+1)}du + \int \frac{-\frac{1}{2}}{(u-1)}du \\ &= \frac{1}{2}(\ln|u+1| - \ln|u-1|) + C = \frac{1}{2}\ln|\frac{u+1}{u-1}| + C \\ &= \frac{1}{2}\ln|\frac{e^x+1}{e^x-1}| + C \end{aligned}$$