QUIZ
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Quiz 11

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Improper Integrals

Determine whether each integral is convergent or divergent.
Evaluate those are convergent.

(6 points) (a) $\int_0^1 \frac{1}{\sqrt{x}} dx$

solution:

convergent

$$\begin{aligned} \int_0^1 \frac{1}{\sqrt{x}} dx &= \lim_{t\to 0^+} \int_t^1 \frac{1}{\sqrt{x}} dx \\ &= \lim_{t\to 0^+} [2\sqrt{x}]|_t^1 = \lim_{t\to 0^+} (2 - 2\sqrt{t}) \\ &= 2 - \lim_{t\to 0^+} 2\sqrt{t} = 2 \end{aligned}$$

(6 points) (b) $\int_1^5 \frac{1}{(5-x)^2} dx$

solution:

divergent

$$\begin{aligned} \int_1^5 \frac{1}{(5-x)^2} dx &= \lim_{t\to 5^-} \int_1^t \frac{1}{(5-x)^2} dx \\ &= \lim_{t\to 5^-} [\frac{1}{5-x}]|_1^t = \lim_{t\to 5^-} (\frac{1}{5-t} - \frac{1}{4}) \\ &= \lim_{t\to 5^-} \frac{1}{5-t} - \frac{1}{4} = \infty \end{aligned}$$

Problem 2: Arc Length

(8 points)Find the length of the curve:
$y = \ln(x^2 - 1)$, $2 \leq x \leq 5$

solution:

By chain rule, we have

$$\frac{dy}{dx} = \frac{2x}{x^2 - 1}.$$

Then, we can obtain the length by Arc Length Theorem

$$\begin{aligned}
L &= \int_2^5 \sqrt{1 + (\frac{dy}{dx})^2} dx \\
&= \int_2^5 \sqrt{1 + (\frac{2x}{x^2 - 1})^2} dx = \int_2^5 \sqrt{1 + \frac{4x^2}{(x^2 - 1)^2}} dx \\
&= \int_2^5 \sqrt{\frac{(x^2 - 1)^2 + 4x^2}{(x^2 - 1)^2}} dx = \int_2^5 \sqrt{\frac{(x^4 - 2x^2 + 1) + 4x^2}{(x^2 - 1)^2}} dx \\
&= \int_2^5 \sqrt{\frac{(x^4 + 2x^2 + 1)}{(x^2 - 1)^2}} dx = \int_2^5 \sqrt{\frac{(x^2 + 1)^2}{(x^2 - 1)^2}} dx \\
&= \int_2^5 \frac{x^2 + 1}{x^2 - 1} dx = \int_2^5 1 + \frac{2}{x^2 - 1} dx \\
&= \int_2^5 1 + \frac{1}{x - 1} - \frac{1}{x + 1} dx = [x + \ln(\frac{x - 1}{x + 1})]|_2^5 \\
&= 5 + \ln(\frac{4}{6}) - 2 - \ln(\frac{1}{3}) \\
&= 3 + \ln(2)
\end{aligned}$$