Quiz 12

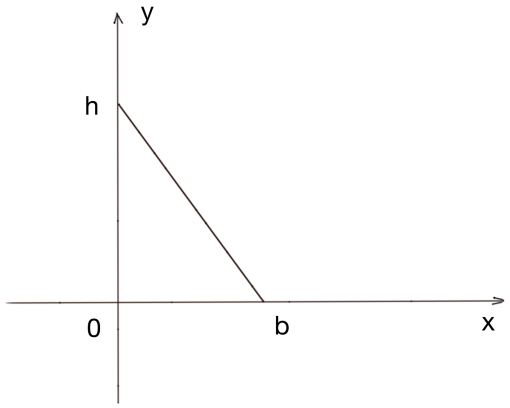
Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Moments and centers of Mass

Consider the triangular lamina with density $\rho = 1$ shown below.



Use calculus to find the following:
(7 points)(a) The moment M_y of the lamina about the $y-$axis.
Hint: $M_y = \rho \int_a^b x f(x) dx$
solution:
Based on the figure, we know

$$f(x) = -\frac{h}{b}x + h.$$

Then, following the hint,

$$\begin{aligned} M_y &= \rho \int_a^b x f(x) dx \\ &= \int_0^b x(-\frac{h}{b}x + h)dx = \int_0^b (-\frac{h}{b}x^2 + hx)dx \\ &= -\frac{h}{3b}b^3 + \frac{h}{2}b^2 \\ &= \frac{hb^2}{6} \end{aligned}$$

(7 points)(b) The moment M_x of the lamina about the $x-$axis.
Hint: $M_x = \rho \int_a^b \frac{1}{2}[f(x)]^2 dx$

$$\begin{aligned} M_x &= \rho \int_a^b \frac{1}{2}[f(x)]^2 dx \\ &= \int_0^b \frac{1}{2}(-\frac{h}{b}x + h)^2 dx = \frac{1}{2}\int_0^b (\frac{h}{b}x - h)^2 dx \\ &= \frac{1}{2}\frac{h^2}{b^2}\int_0^b (x-b)^2 dx = \frac{1}{2}\frac{h^2}{b^2}\frac{1}{3}(x-b)^3|_0^b \\ &= \frac{1}{6}\frac{h^2}{b^2}(0-(-b)^3) \\ &= \frac{h^2 b}{6} \end{aligned}$$

(6 points)(c) The center of mass $(\bar{x}, \bar{y})$ of the lamina.

$$M = \rho A = 1 * \frac{1}{2}hb = \frac{1}{2}hb$$

We have $(\bar{x}, \bar{y}) = (\frac{M_y}{M}, \frac{M_x}{M}) = (\frac{b}{3}, \frac{h}{3})$