

Quiz 13

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without necessary explanation.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Series

Determine whether the series is convergent or divergent.

If it is convergent, find its sum.

You must justify your answer.

(a) $\sum_{n=1}^{\infty} \frac{n}{n+1}$

solution:

$$a_n = \frac{n}{n+1} \text{ and } \lim_{n \rightarrow \infty} a_n = 1 \neq 0$$

Thus, series diverges by Test for Divergence.

(b) $\sum_{n=1}^{\infty} \frac{5^n}{3^{n+1}}$

solution:

Method 1

$$a_n = \frac{5^n}{3^{n+1}} \text{ and } \lim_{n \rightarrow \infty} a_n = \infty \neq 0$$

Thus, series diverges by Test for Divergence.

Method 2

$$\sum_{n=1}^{\infty} \frac{5^n}{3^{n+1}} = \frac{5}{3^2} + \frac{5^2}{3^3} + \frac{5^3}{3^4} + \cdots = \frac{5}{3^2} [1 + \frac{5}{3} + (\frac{5}{3})^2 + \cdots]$$

Geometric series with $|r| = \frac{5}{3} > 1$. Thus, series diverges.

(c) $\sum_{n=1}^{\infty} (-\frac{1}{3})^{n-1}$,

solution:

$$\sum_{n=1}^{\infty} (-\frac{1}{3})^{n-1} = 1 + (-\frac{1}{3}) + (-\frac{1}{3})^2 + \cdots = \frac{1}{1 - (-\frac{1}{3})} = \frac{3}{4}$$

Geometric series with $|r| = \frac{1}{3} < 1$. Thus, series converges.

(d) $\sum_{n=1}^{\infty} \frac{7}{4^{n+2}}$

solution:

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{7}{4^{n+2}} &= \frac{7}{4^3} + \frac{7}{4^4} + \frac{7}{4^5} + \cdots \\ &= \frac{7}{4^3} \left[1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \cdots \right] \\ &= \frac{7}{4^3} \frac{1}{1 - \frac{1}{4}} = \frac{7}{48} \end{aligned}$$

Geometric series with $r = \frac{1}{4}$, $|r| = \frac{1}{4} < 1$