QUIZ
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Quiz 16

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Ratio Test

Use the Ratio Test to determine whether the series is convergent or divergent.
You must evaluate the limit $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}|$

(6 points)(I) $\sum_{n=1}^{\infty} \frac{n}{5^n}$,

Solution: Converge

We have $a_n = \frac{n}{5^n}$, then

$$\begin{aligned} \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| &= \lim_{n\to\infty} \frac{n+1}{5^{n+1}} \cdot \frac{5^n}{n} \\ &= \lim_{n\to\infty} \frac{n+1}{5n} = \frac{1}{5} < 1 \end{aligned}$$

Thus, by root test, this series converges.

(7 points)(II) $\sum_{n=1}^{\infty} \frac{n!}{96^n}$,

Solution: Diverge

We have $a_n = \frac{n!}{96^n}$, then

$$\begin{aligned} \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| &= \lim_{n\to\infty} \frac{(n+1)!}{96^{n+1}} \cdot \frac{96^n}{n!} \\ &= \lim_{n\to\infty} \frac{n+1}{96} = \infty > 1 \end{aligned}$$

Thus, by root test, this series diverges.

(7 points)(III) $\sum_{n=1}^{\infty} \frac{11^n}{(n+4)8^{2n+3}}$

Solution: Converge

We have $a_n = \frac{11^n}{(n+4)8^{2n+3}}$, then

$$\begin{aligned} \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| &= \lim_{n\to\infty} \frac{11^{n+1}}{(n+5)8^{2n+5}} \cdot \frac{(n+4)8^{2n+3}}{11^n} \\ &= \lim_{n\to\infty} \frac{11}{8^2} \cdot \frac{n+4}{n+5} = \frac{11}{64} < 1 \end{aligned}$$

Thus, by root test, this series converges.