Quiz 17

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Ratio and Root Test

(6 points) The series $\sum\limits_{n=1}^{\infty} \dfrac{(8n+9)^n}{n^{3n}}$ converges by the root test.
True or False? (No reason needed)
solution:
True
We can see that
$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{(8n+9)}{n^3} = 0 < 1.$$
Thus, the series converges.

Problem 2: General Strategy

(14 points) Test the series for convergence or divergence
You must state clearly what test you are using and verify
that the conditions of the test are satisfied.
(I) $\sum\limits_{n=1}^{\infty} \dfrac{(2n)!}{e^{n^2}}$,
solution:
Converge
Identify $a_n = \dfrac{(2n)!}{e^{n^2}}$, and evaluate the limit

$$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \lim_{n\to\infty} |\frac{(2n+2)!}{e^{(n+1)^2}} \cdot \frac{e^{n^2}}{(2n)!}| = \lim_{n\to\infty} |\frac{(2n+1)(2n+2)}{e^{2n+1}}| = 0 < 1.$$

Thus, the series converges.

(II) $\sum\limits_{n=1}^{\infty} \sin(\dfrac{3}{n^2})$
solution:
Converge
By limit comparison test with series $\sum\limits_{n=1}^{\infty} \dfrac{3}{n^2}$, we know that the series converges.