QUIZ
Xinyu Liu
March 31, 2018

Quiz 18

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Power Series

(8 points) Find the radius of convergence and
the interval of convergence of the power series.
$\sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n}5^n}$
solution:
Identify $a_n = \dfrac{(x-2)^n}{\sqrt{n}5^n}$ and evaluate the limit

$$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = \lim_{n\to\infty} |\frac{(x-2)^{n+1}}{\sqrt{n+1}5^{n+1}} \cdot \frac{\sqrt{n}5^n}{(x-2)^n}| = \lim_{n\to\infty} |\frac{x-2}{5} \cdot \sqrt{\frac{n}{n+1}}| = \frac{|x-2|}{5} < 1$$

We have $|x-2| < 5$ which means $R = 5$.
Check the endpoints $x_1 = -3$ and $x_2 = 7$, we have $I = [-3, 7)$.

Problem 2: Functions and Power Series

(12 Points) Find a power series representation
for the function and give the radius of convergence.
(Give your power series representation centred at $x = 0$.)
(I)$f(x) = \ln(9-x)$,
solution:

$$\begin{aligned} \ln(9-x) &= \int \frac{1}{x-9}dx = -\frac{1}{9}\int \frac{1}{1-\frac{x}{9}}dx \\ &= -\frac{1}{9}\int \sum_{n=0}^{\infty}(\frac{x}{9})^n dx = C - \frac{1}{9}\int \sum_{n=0}^{\infty} \frac{x^{n+1}}{n9^n}dx \\ &= C - \sum_{n=1}^{\infty} \frac{x^n}{n9^n} \end{aligned}$$

Putting $x = 0$, we get $C = \ln 9$, the series coverge for $|\dfrac{x}{9}| < 1$ which means $|x| < 9$, so $R = 9$.
(II)$f(x) = \frac{x}{4+x^2}$.

solution:

$$
\begin{aligned}
f(x) &= \frac{x}{4+x^2} = \frac{x}{4}\cdot(\frac{1}{1+\frac{x^2}{4}}) = \frac{x}{4}\cdot[\frac{1}{1-(-\frac{x^2}{4})}] \\
&= \frac{x}{4}\cdot\sum_{n=0}^{\infty}(-\frac{x^2}{4})^n = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{4^{n+1}}
\end{aligned}
$$

The series coverge for $|\frac{x^2}{4}| < 1$ which means $|x| < 2$, so $R = 2$.