QUIZ
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January 18, 2018

Quiz 1

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

Problem 0: Quiz checklist

Please write the section number and your name.

Problem 1: Vector

Let $\overrightarrow{a} = 2\overrightarrow{i} + 2\overrightarrow{j} - \overrightarrow{k}$.
(5 points)(a) Find a unit vector in the direction of $\overrightarrow{a}$

Solution
Step 1:

$$\overrightarrow{a} =< 2, 2, -1 >$$

Step 2:

$$|\overrightarrow{a}| = \sqrt{2^2 + 2^2 + (-1)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

Step 3:
The unit vector is

$$\overrightarrow{u} = \frac{\overrightarrow{a}}{|\overrightarrow{a}|} = \frac{1}{3} < 2, 2, -1 >=< \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} >$$

(5 points)(b) Find a vector in the <u>**opposite**</u> direction $\overrightarrow{a}$ of length 10.

$$\overrightarrow{b} = -10\overrightarrow{u} =< -\frac{20}{3}, -\frac{20}{3}, \frac{10}{3} >$$

Problem 2: Equation of sphere

(10 points) Find the center and radius of the sphere with equation

$$2x^2 + 2y^2 + 2z^2 = 4x + 2y - 6z$$

Solution

$$x^2 + y^2 + z^2 = 2x + y - 3z$$
$$x^2 + y^2 + z^2 - 2x - y + 3z = 0$$

Method 1:

$$x^2 - 2x + y^2 - y + z^2 + 3z = 0$$
$$x^2 - 2x + 1 + y^2 - y + z^2 + 3z = 1$$
$$(x-1)^2 + y^2 - y + \frac{1}{4} + z^2 + 3z = 1 + \frac{1}{4}$$
$$(x-1)^2 + (y - \frac{1}{2})^2 + z^2 + 3z + \frac{9}{4} = \frac{5}{4} + \frac{9}{4}$$
$$(x-1)^2 + (y - \frac{1}{2})^2 + (z + \frac{3}{2})^2 = \frac{14}{4} = \frac{7}{2}$$

Thus, center is $(1, \frac{1}{2}, -\frac{3}{2})$ and $r = \sqrt{\frac{7}{2}}$.

Method2:

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$
$$x^2 - 2ax + a^2 + y^2 - 2by + b^2 + z^2 - 2cz + c^2 = r^2$$
$$x^2 + y^2 + z^2 - 2ax - 2by - 2cz = r^2 - a^2 - b^2 - c^2$$
$$x^2 + y^2 + z^2 - 2x - y + 3z = 0$$
$$-2a = -2, -2b = -1, -2c = 3, r = \sqrt{a^2 + b^2 + c^2}$$

Thus, $a = 1, b = \frac{1}{2}, c = -\frac{3}{2}$ and $r = \sqrt{1^2 + (\frac{1}{2})^2 + (-\frac{3}{2})^2} = \sqrt{1 + \frac{1}{4} + \frac{9}{4}} = \sqrt{\frac{14}{4}} = \sqrt{\frac{7}{2}}$.