purdue university · MA 16200
Calculus II

QUIZ
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Quiz 2

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Dot product

Let $\overrightarrow{a} = 2\overrightarrow{i} + 2\overrightarrow{j} + \overrightarrow{k}$, $\overrightarrow{b} = \overrightarrow{i} + 2\overrightarrow{j} - 3\overrightarrow{k}$. Find the follwoing:
(3 points)(a) $\overrightarrow{a} \cdot \overrightarrow{b}$

$$\overrightarrow{a} = (2, 2, 1), \overrightarrow{b} = (1, 2, -3).$$
$$\overrightarrow{a} \cdot \overrightarrow{b} = 2 \cdot 1 + 2 \cdot 2 + 1 \cdot (-3) = 2 + 4 - 3 = 3$$

(3 points)(b) The cosine angle between the $\overrightarrow{a}$ and $\overrightarrow{b}$

$$|\overrightarrow{a}| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$$
$$|\overrightarrow{b}| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$
$$\cos(\theta) = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}||\overrightarrow{b}|} = \frac{3}{3 \cdot \sqrt{14}} = \frac{1}{\sqrt{14}}$$

Thus,$\theta = \arccos^{-1}(\frac{1}{\sqrt{14}})$
(3 points)(c) The scalar projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$: $comp_{\overrightarrow{a}}\overrightarrow{b}$

$$comp_{\overrightarrow{a}}\overrightarrow{b} = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}|} = \frac{3}{3} = 1$$

(3 points)(d) The vector projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$: $proj_{\overrightarrow{a}}\overrightarrow{b}$

$$proj_{\overrightarrow{a}}\overrightarrow{b} = comp_{\overrightarrow{a}}\overrightarrow{b} \cdot \overrightarrow{u}_a = 1 \cdot \frac{(2, 2, 1)}{3} = (\frac{2}{3}, \frac{2}{3}, \frac{1}{3})$$

Problem 2: Property of dot product

(8 points) Let $|\overrightarrow{a}| = 3$, $|\overrightarrow{b}| = 2$ and $\overrightarrow{a} \cdot \overrightarrow{b} = 6$. Are $\overrightarrow{a}$ and $\overrightarrow{b}$ parallel, orthongonal or neither? Why?

Parallel.

$$\cos(\theta) = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}||\overrightarrow{b}|} = \frac{6}{2 \cdot 3} = 1$$

Thus, $\theta = 0$