QUIZ
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Quiz 3

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Find vector by cross product

(10 points)Find two vectors perpendicular to the plane through the points $P(1,1,1)$, $Q(2,-1,4)$ and $R(1,-2,-3)$.

Step 1: Convert the points to vectors
$\overrightarrow{PR} = (1,-2,-3) - (1,1,1) = (0,-3,-4)$,
$\overrightarrow{PQ} = (2,-1,4) - (1,1,1) = (1,-2,3)$.

Step 2: Compute the determinant

$$\begin{aligned}
\mathbf{a} &= \overrightarrow{PR} \times \overrightarrow{PQ} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -3 & -4 \\ 1 & -2 & 3 \end{vmatrix} \\
&= \begin{vmatrix} -3 & -4 \\ -2 & 3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 0 & -4 \\ 1 & 3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 0 & -3 \\ 1 & -2 \end{vmatrix} \mathbf{k} \\
&= -17\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}
\end{aligned}$$

Step 2: Consider the opposite direction by multiplying -1.

$$\mathbf{b} = -\mathbf{a} = -(-17\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) = 17\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$$

Thus, there are two vectors $\mathbf{a} = (-17,-4,3)$ and $\mathbf{b} = (17,4,-3)$

Problem 2: Find area by cross product

(10 points) Find the area of triangle PQR: $P(1,-1)$, $Q(0,-2)$ and $R(3,0)$.

Step 1: Convert the points to vectors
$\overrightarrow{PR} = (3,0) - (1,-1) = (2,1)$
$\overrightarrow{PQ} = (0.-2) - (1,-1) = (-1,-1)$.

Step 2: Vectors in $\mathbb{R}^3$ defining the triangle by $\overrightarrow{PR} = (2,1,0)$
$\overrightarrow{PQ} = (-1,-1,0)$.

Step 3:

Compute the determinant

$$\begin{aligned}
\mathbf{a} &= \overrightarrow{PR} \times \overrightarrow{PQ} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 0 \\ -1 & -1 & 0 \end{vmatrix} \\
&= \begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & 0 \\ -1 & 0 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} \mathbf{k} \\
&= 0\mathbf{i} + 0\mathbf{j} - 1\mathbf{k}
\end{aligned}$$

Step 4: We know the area of the parallelogram is $|(0,0,-1)| = 1$, and the area of triangle is half of it.

$$A = \frac{1}{2}$$

Method 2:

Step 1: Convert the points to vectors
$\overrightarrow{PR} = (3,0) - (1,-1) = (2,1)$
$\overrightarrow{PQ} = (0.-2) - (1,-1) = (-1,-1)$.

Step 2: Compute the absolute value of this determinate

$$\begin{aligned}
A_1 &= \left| \begin{vmatrix} \overrightarrow{PR} \\ \overrightarrow{PQ} \end{vmatrix} \right| \\
&= \left| \begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} \right| \\
&= |-2-(-1)| \\
&= 1
\end{aligned}$$

Step 3: This value is area of the parallelogram, so the area A_2 of triangle PQR is A_1 divided by 2.

$$A_2 = \frac{1}{2} A_1 = \frac{1}{2}$$