

Quiz 4

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without necessary explanation.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

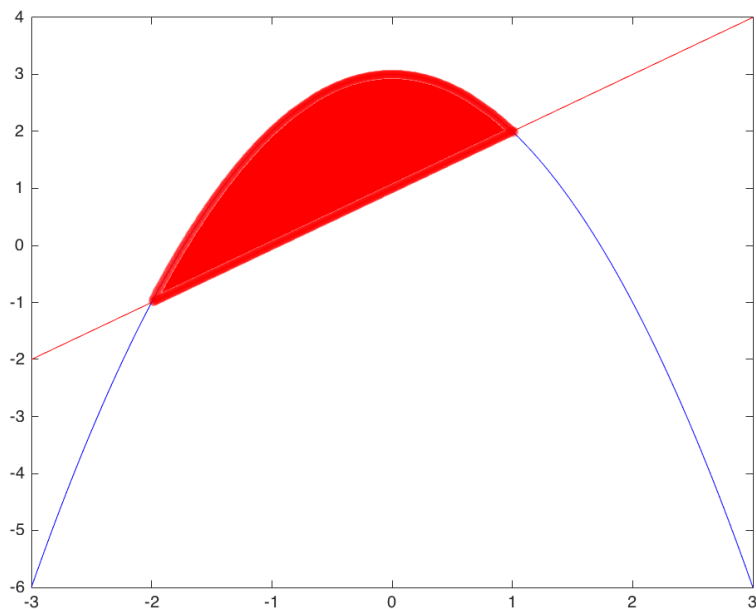
Problem 1: Area between curves

(5 points) (a) Sketch the region enclosed by the curves

$$y = x + 1 \text{ and } y = 3 - x^2,$$

and set up a definite integral that gives the area of the region. **Do not evaluate the integral.**

solution: Find intersection points



$$x + 1 = 3 - x^2$$

Thus, $x = 1$ or -2 . Then, we have

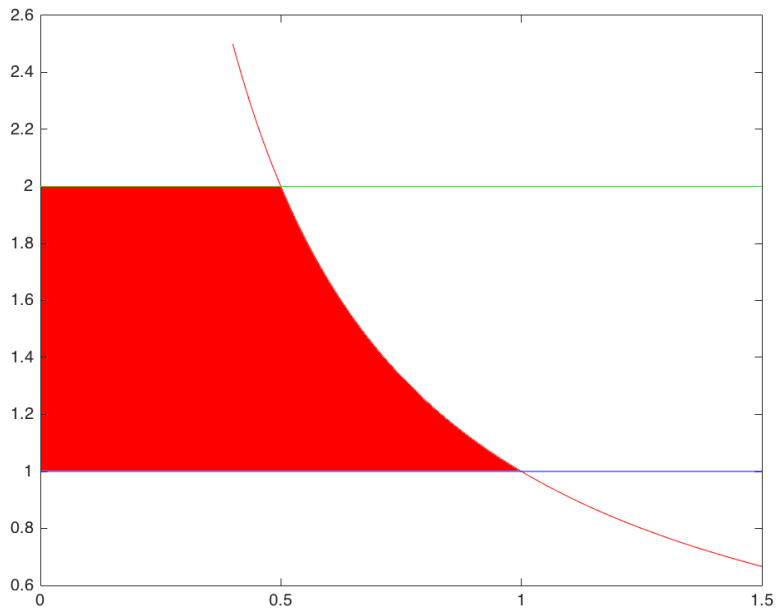
$$\int_{-2}^1 y_T - y_B = \int_{-2}^1 (3 - x^2) - (x + 1) dx$$

(5 points) (b) Sketch the region enclosed by the curves

$$y = \frac{1}{x}, x = 0, y = 1, y = 2$$

and find the area of the region.

solution:



We consider this integration with respect to y .

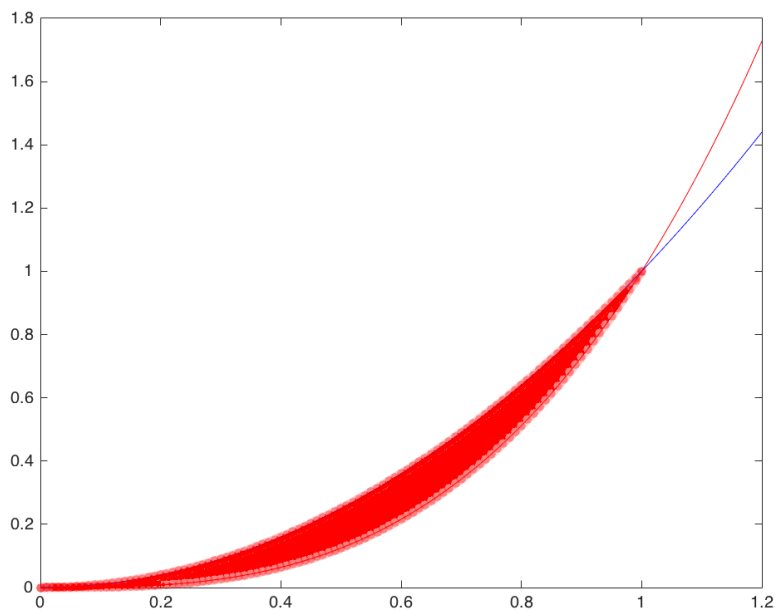
We have $x_R = \frac{1}{y}$ and $x_L = 0$.

$$\int_1^2 x_R - x_L = \int_1^2 \frac{1}{y} dy = \ln(y) \Big|_1^2 = \ln(2)$$

Problem 2: Volume

(10 points) Find the volume V of the solid obtained by rotating the region enclosed by the curves $y = x^2$ and $y = x^3$ about the x -axis.

solution:



Using washer method, we have outer radius $R = x^2$ and inner radius $r = x^3$. Then, we have

$$\int_0^1 \pi R^2 - \pi r^2 dx = \pi \int_0^1 x^4 - x^6 dx = \pi \left(\frac{1}{5} - \frac{1}{7} \right) = \frac{2\pi}{35}$$