

Quiz 5

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without necessary explanation.)

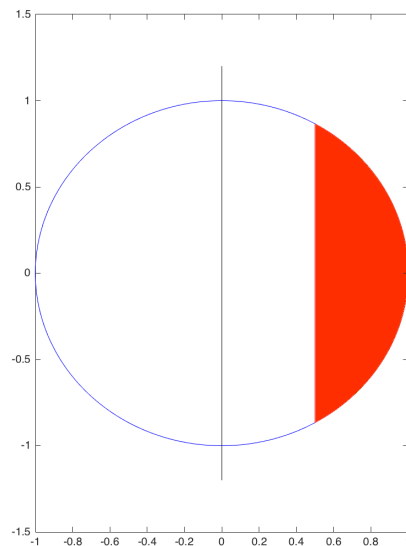
Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Cylindrical shells

(10 points) The region inside the circle $x^2 + y^2 = 1$ and to the right of the line $x = \frac{1}{2}$ is rotated about the y -axis. Use the method of shells to find the volume of the resulting solid.

solution:



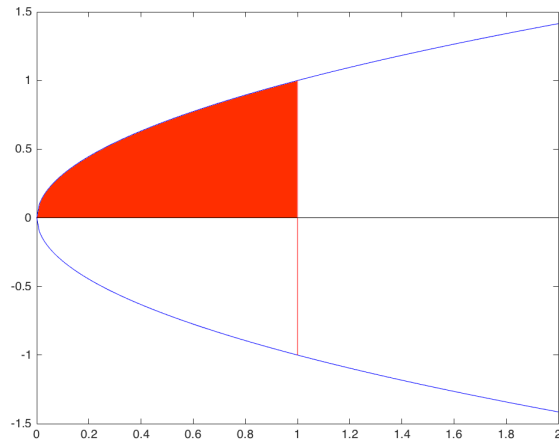
Using shells method, we have height $h = 2\sqrt{1 - x^2}$ and radius $r = x$. From the figure, we can see the interval is from $a = \frac{1}{2}$ to $b = 1$. Then, we have

$$\int_a^b 2\pi r h dx = \int_{\frac{1}{2}}^1 2\pi x (2\sqrt{1 - x^2}) dx = \frac{\sqrt{3}}{2} \pi$$

Problem 2: Comparison of different methods

Set up, but **do not evaluate**, an integral to find the volume obtained by rotating the region bounded by $y = 0$, $x = y^2$, and $x = 1$ about the y -axis using:

solution:



(5 points)(a) cylindrical shells

Using shells method, we have height $h = \sqrt{x}$ and radius $r = x$. From the figure, we can see the interval is from $a = 0$ to $b = 1$. Then, we have

$$\int_a^b 2\pi r h dx = \int_0^1 2\pi x(\sqrt{x}) dx = \frac{4}{5}\pi$$

(5 points)(b) disks/washers

Using washers method, we have outer radius $R = 1$ and inner radius $r = x = y^2$. From the figure, we can see the interval is from $a = 0$ to $b = 1$. Then, we have

$$\int_a^b \pi R^2 - \pi r^2 dy = \int_0^1 \pi(1^2 - (y^2)^2) dy = \frac{4}{5}\pi$$