

## Quiz 6

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without necessary explanation.)

### Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

### Problem 1: Work

(10 points) A force of 50 N is required to hold a spring that has been stretched from its natural length of 1 m to a length of 1.5 m. How much work is done (in joules = N·m) by stretching the spring from a length of 2 m to a length of 3 m?

By the Hooke's Law, we have

$$F = k\Delta x$$

Given by the problem,  $F = 50$  N and  $\Delta x = 1.5 - 1 = 0.5$  m. Thus,

$$k = \frac{F}{\Delta x} = \frac{50}{0.5} = 100 \text{ N/m.}$$

Then,

$$W = \int_2^3 k\Delta x dx = \int_2^3 k(x-1)dx = \frac{1}{2}k(x-1)^2 \Big|_{x=2}^{x=3} = 150 \text{ J}$$

### Problem 2: Integration by Parts

(5 points) (a) Evaluate  $\int \arctan\left(\frac{1}{x}\right)dx$  Let  $u = \arctan\left(\frac{1}{x}\right)$  and  $v = x$ , then

$$\begin{aligned} du &= \frac{1}{1 + \left(\frac{1}{x}\right)^2} \left(-\frac{1}{x^2}\right) dx = -\frac{1}{1 + x^2} dx \\ dv &= dx \end{aligned}$$

$$\begin{aligned} \int \arctan\left(\frac{1}{x}\right) dx &= \arctan\left(\frac{1}{x}\right)x - \int x \left(-\frac{1}{1+x^2}\right) dx + C \\ &= \arctan\left(\frac{1}{x}\right)x + \int \frac{1}{1+x^2} \cdot \frac{1}{2} 2x dx + C \\ &= \arctan\left(\frac{1}{x}\right)x + \frac{1}{2} \int \frac{1}{u} du + C \\ &= \arctan\left(\frac{1}{x}\right)x + \frac{1}{2} \ln|1+x^2| + C \end{aligned}$$

(5 points) (b) Evaluate  $\int e^x \sin(2x) dx$

$$\begin{aligned}\int e^x \sin(2x) dx &= e^x \sin(2x) - \int e^x (2 \cos(2x)) dx + C \\ &= e^x \sin(2x) - 2[e^x \cos(2x) - \int e^x (-2 \sin(2x)) dx] + C \\ &= e^x \sin(2x) - 2e^x \cos(2x) - 4 \int e^x \sin(2x) dx + C \\ \int e^x \sin(2x) dx &= \frac{1}{5}(e^x \sin(2x) - 2e^x \cos(2x)) + C\end{aligned}$$