# Quiz 7

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

## Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

## Problem 1: Trigonometric Integrals

Evaluate the integrals:

(6 points) (a) $\int \sin^4(x) \cos^3(x) dx$

**solution**:

$$
\begin{aligned}
I &= \int \sin^4(x) \cos^2(x) \cos(x) dx \\
&= \int \sin^4(x)(1 - \sin^2(x)) \cos(x) dx \quad (\text{use } \cos^2(x) = 1 - \sin^2(x)) \\
&= \int u^4(1 - u^2) du \quad (\text{use } u = \sin(x)) \\
&= \int u^4 - u^6 du \\
&= \frac{u^5}{5} - \frac{u^7}{7} + C \\
&= \frac{\sin^5(x)}{5} - \frac{\sin^7(x)}{7} + C
\end{aligned}
$$

(7 points) (b) $\int_0^{\frac{\pi}{2}} \cos^2(x) dx$

**solution**:

$$
\begin{aligned}
I &= \int_0^{\frac{\pi}{2}} \frac{1}{2}(\cos(2x) + 1) dx \quad (\text{use } \cos^2(x) = \frac{\cos(2x) + 1}{2}) \\
&= \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos(2x) dx + \frac{1}{2} \int_0^{\frac{\pi}{2}} 1 dx \\
&= \frac{1}{2} \int_0^{\pi} \cos(t) \frac{1}{2} dt + \frac{1}{2} x \big|_0^{\frac{\pi}{2}} \quad (\text{use } t = 2x) \\
&= \sin(t) \big|_0^{\pi} + \frac{\pi}{4} \\
&= \frac{\pi}{4}
\end{aligned}
$$

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(7 points) (c) $\int \tan^3(x) \sec(x) dx$

**solution**:

$$
\begin{aligned}
I &= \int \tan^2(x) \tan(x) \sec(x) dx \\
&= \int (\sec^2(x) - 1) \tan(x) \sec(x) dx \quad (\text{use } \tan^2(x) = \sec^2(x) - 1) \\
&= \int (u^2(x) - 1) du \quad (\text{use } u = \sec(x)) \\
&= \frac{u^3}{3} - u + C
\end{aligned}
$$