Quiz 8

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without nessary explaination.)

Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**.

Problem 1: Trigonometric Substitution

Evaluate the integrals:

(8 points) (a) $\int \frac{1}{x^2\sqrt{25-x^2}}dx$

solution:

$$
\begin{aligned}
I &= \int \frac{1}{(5\sin(\theta))^2 5\cos(\theta)} 5\cos(\theta)d\theta \ (\text{Use } x = 5\sin(\theta)) \\
&= \frac{1}{25}\int \frac{1}{\sin^2(\theta)}d\theta = \frac{1}{25}\int \csc^2(\theta)d\theta \\
&= -\frac{1}{25}\cot(\theta) + C \\
&= -\frac{1}{25}\frac{\cos(\theta)}{\sin(\theta)} + C \\
&= -\frac{1}{25}\frac{\sqrt{25-x^2}}{x} + C
\end{aligned}
$$

(12 points) (b) $\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{x^2+4}}dx$

solution:

$$
\begin{aligned}
I &= \int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{x^2+4}}dx = \int_0^{\frac{\pi}{3}} \frac{(2\tan(\theta))^3}{2\sec(\theta)}2\sec^2(\theta)d\theta \ (\text{Use } x = 2\tan(\theta)) \\
&= \int_0^{\frac{\pi}{3}} 8\tan^3(\theta)\sec(\theta)d\theta = \int_0^{\frac{\pi}{3}} 8\tan^2(\theta)\cdot\tan(\theta)\sec(\theta)d\theta \\
&= 8\int_0^{\frac{\pi}{3}} (\sec^2(\theta)-1)\cdot\tan(\theta)\sec(\theta)d\theta = 8\int_1^2 (u^2-1)du \ (\text{Use } u = \sec(\theta)) \\
&= 8[\frac{u^3}{3}-u]|_1^2 \\
&= \frac{32}{3}
\end{aligned}
$$

Sorry for the typo, and if you do the integration $\int_0^{2\sqrt{3}} \frac{x^3}{x^2+4} dx$, please see this answer

solution:

$$
\begin{aligned}
I &= \int_0^{2\sqrt{3}} \frac{x^3}{x^2+4} dx = \int_0^{2\sqrt{3}} \frac{x^2}{x^2+4} x dx \\
&= \int_0^{12} \frac{u}{u+4} \frac{1}{2} du \ (\text{Use } u = x^2) \\
&= \frac{1}{2} \int_0^{12} \frac{u}{u+4} du = \frac{1}{2} \int_0^{12} \frac{u+4-4}{u+4} du = \frac{1}{2} \int_0^{12} 1 - \frac{4}{u+4} du \\
&= \frac{1}{2} \int_0^{12} 1 - \frac{4}{u+4} du \\
&= \frac{1}{2} [u - 4\ln|u+4|]|_0^{12} \\
&= \frac{1}{2} [(12 - 4\ln(16)) - (0 - 4\ln(4)) \\
&= \frac{1}{2} [12 - 4\ln(4)] = \frac{1}{2} [12 - 2\ln(4)] \\
&= 6 - 4\ln(2)
\end{aligned}
$$