

## Quiz 9

Please answer the following questions in complete sentences in a clearly prepared manuscript. (No credits for the answer without necessary explanation.)



### Problem 0: Quiz checklist

Please write the section number, your name and special number on the **back**. Since today is the lunar new year, you will get 10 points automatically when you turn in your quiz.

### Problem 1: Trigonometric Substitution

(5 points) Evaluate the integrals:

$$\int_0^1 \frac{1}{(x^2 + 1)^{\frac{3}{2}}} dx$$

**solution:**

$$\begin{aligned}
I &= \int_0^{\frac{\pi}{4}} \frac{1}{(\tan^2(\theta) + 1)^{\frac{3}{2}}} \sec^2(\theta) d\theta \quad (\text{Use } x = \tan(\theta)) \\
&= \int_0^{\frac{\pi}{4}} \frac{1}{\sec^3(\theta)} \sec^2(\theta) d\theta = \int_0^{\frac{\pi}{4}} \frac{1}{\sec(\theta)} d\theta \\
&= \int_0^{\frac{\pi}{4}} \cos(\theta) d\theta = \sin(\theta) \Big|_0^{\frac{\pi}{4}} \\
&= \frac{\sqrt{2}}{2}
\end{aligned}$$

## Problem 2: Integration by Partial Fractions

(5 points) Evaluate the integrals:

$$\int \frac{x}{x^2 + 6x + 9} dx$$

**solution:** Since  $x^2 + 6x + 9 = (x + 3)^2$ , we have

$$\frac{x}{x^2 + 6x + 9} = \frac{A}{x + 3} + \frac{B}{(x + 3)^2}$$

Multiply both sides by  $(x + 3)^2$ , we get

$$x = A(x + 3) + B \tag{1}$$

Substituting  $-3$  for  $x$  gives  $B = -3$ . Since coefficients of the  $x$ -terms in equation (1) must be equal, so we have  $A = 1$ .

$$\begin{aligned}
\int \frac{x}{x^2 + 6x + 9} dx &= \int \frac{1}{x + 3} + \frac{-3}{(x + 3)^2} dx \\
&= \int \frac{1}{x + 3} - 3 \int \frac{1}{(x + 3)^2} dx \\
&= \ln|x + 3| + \frac{1}{x + 3} + C
\end{aligned}$$