Review for Exam 2

Notice: This is only a quick review for exam 2. It does not cover all the content of exam 2.

1. You are supposed to know how to compute the integration of the form:

$$\int \sin^m(x)\cos^n(x)dx \text{ and } \int \tan^m(x)\sec^n(x)dx$$

Example Problems

Compute the following integrals:

$\int \sin^3(x)\cos^3(x)dx$, $\int \sin^4(x)\cos^2(x)dx$, $\int \sec^4(x)\tan^2(x)dx$,

$\int \sec^3(x)\tan(x)dx$, $\int \sec^3(x)dx$, $\int \sec(x)dx$.

2. You are supposed to know how to use the 3 typies of trig substitution, and carry out the integration accordingly.

(1)$\sqrt{a^2-x^2}$, $u = a\sin(\theta)$, $du = a\cos(\theta)$, $\sqrt{a^2-x^2} = a\cos(\theta)$,

(2)$\sqrt{a^2+x^2}$, $u = a\tan(\theta)$, $du = a\sec^2(\theta)$, $\sqrt{a^2+x^2} = a\sec(\theta)$,

(3)$\sqrt{x^2-a^2}$, $u = a\sec(\theta)$, $du = a\tan(\theta)\sec(\theta)$, $\sqrt{x^2-a^2} = a\tan(\theta)$,

Example Problems

Compute the following integrals:

$\int \sqrt{7x^2+1}dx$, $\int \frac{x}{\sqrt{3-2x-x^2}}dx$, $\int \sqrt{5-4x^2}dx$.

3. You are supposed to know

(1) the proper form of the partial fractions,

(2) how to determine the appropriate constants apperaring in the partial fraction,

(3) how to compute the integral accordingly.

Example Problems

Compute the following integrals:

$\int \frac{x^2}{(x-1)^2}dx$, $\int \frac{x+2}{x^2+2x+2}dx$, $\int \frac{x}{(x+1)(x-1)(x-2)}dx$, $\int \frac{x^2}{(x-1)^2(x^2+1)}$.

4. You are supposed to know how to compute the arc length by the formula

$$L = \int_a^b \sqrt{1+(\frac{dy}{dx})^2}dx = \int_c^d \sqrt{1+(\frac{dx}{dy})^2}dy.$$

Example Problems

Problem 9 and 11 in section 8.1 on page 549 of the textbook.

5. You are supposed to know how to compute the area of the surface obtained

(1) by rotating the curve around the $x-$axis

$$S = \int_a^b 2\pi y\sqrt{1+(\frac{dy}{dx})^2}dx = \int_c^d 2\pi y\sqrt{1+(\frac{dx}{dy})^2}dy$$

(2) or, by rotating the curve around the $y-$axis

$$S = \int_a^b 2\pi x \sqrt{1 + (\frac{dy}{dx})^2} dx = \int_c^d 2\pi x \sqrt{1 + (\frac{dx}{dy})^2} dy$$

Example Problems
Example 3 in section 8.2 on page 555 of the textbook.

6. You are supposed to know why a given improper integral is improper, and accordingly to be able to determine if the given improper integral is convergent/divergent. In case it is convergent, you should be able to compute its value.
Example Problems
Evaluate the following improper integrals:
$\int_0^\infty \frac{e^x}{e^{2x}+1} dx$, $\int_1^9 \frac{1}{x-1}$, $\int_1^9 \frac{1}{\sqrt{x-1}}$, $\int_{-\infty}^\infty x dx$, $\int_{-\infty}^\infty x e^{-x^2} dx$.

7. You are supposed to be able to compute the coordinates $(\bar{x}, \bar{y})$ of the centroid of a given figure.

$$\bar{x} = \frac{\int_a^b x[f(x) - g(x)]dx}{\int_a^b [f(x) - g(x)]dx}$$

$$\bar{y} = \frac{\int_a^b \frac{1}{2}[f(x)^2 - g(x)^2]dx}{\int_a^b [f(x) - g(x)]dx}$$

Example Problems
Example 6 in section 8.3 on page 564 of the textbook.

8. You are supposed to be able to determine if a given sequence is convergent/divergent. In case it is convergent, you should be able to compute its limit.
Example Problems
Webassign HW19 Problems 7, 8, 9, 10.