Techniques of Integration

PURDUE
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Integration by Parts

$$\int udv = uv - \int vdu$$

- Why we need integration by part?

Integration by Parts

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- Why we need integration by part?
  To obtain a simpler integral than the one we started with.
- Find $\int x \sin(x) dx$

Integration by Parts

- Use integration by part twice or more.
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  - Find $\int t^2 e^t dt$
- Use integration by part twice or more until come back.
  - Find $\int e^x \sin(x) dx$
- Combine integration by part with trig integration.
  - Find $\int \sec^3(x) dx$

Trigonometric Integrals

- Types of Trig Integration
  - $\int \sin^m(x)\cos^n(x)dx$
  - $\int \tan^m(x)\sec^n(x)dx$
  - $\int \sin(mx)\cos(nx)dx$, $\int \sin(mx)\sin(nx)dx$ and $\int \cos(mx)\cos(nx)dx$

Trigonometric Integrals

- $\int \sin^m(x)\cos^n(x)dx$
  - ***m* odd or *n* odd.** Find $\int \sin^4(x)\cos^5(x)dx$
    Strategy: Extract one $\cos(x)$ and use the substitution $u = \sin(x)$ via the equality $\cos^2(x) = 1 - \sin^2(x)$.
  - ***m* even and *n* even.** Find $\int \sin^2(x)\cos^2(x)dx$
    Strategy: Using the double angle formula to drop the degree.

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

Trigonometric Integrals

- $\int \tan^m(x)\sec^n(x)dx$
  - $\boldsymbol{n \neq 0}$ **even.** Find $\int \tan^3(x)\sec^4(x)dx$
  - $\boldsymbol{n = 0}$**.** Find $\int \tan(x)dx$, $\int \tan^3(x)dx$
  - ***m*** **odd &** ***n*** **odd.** Find $\int \tan^3(x)\sec^5(x)dx$
  - ***m*** **even &** ***n*** **odd.** Find $\int \sec(x)dx$, $\int \sec^3(x)dx$

Trigonometric Integrals

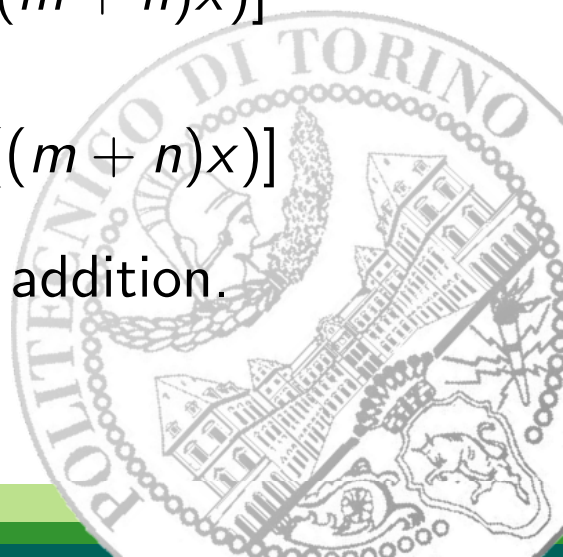
- $\int \sin(mx)\cos(nx)dx$
- $\int \sin(mx)\sin(nx)dx$
- $\int \cos(mx)\cos(nx)dx$ Strategy: Using the formulas accordingly,

$$\sin(mx)\cos(nx) = \frac{1}{2}[\sin((m-n)x) + \sin((m+n)x)]$$

$$\sin(mx)\sin(nx) = \frac{1}{2}[\cos((m-n)x) - \cos((m+n)x)]$$

$$\cos(mx)\cos(nx) = \frac{1}{2}[\cos((m-n)x) + \cos((m+n)x)]$$

  we split the multiplication of sin and cos into the addition.
- Find $\int \sin(4x)\cos(5x)dx$



Trigonometric Substitution

- $\sqrt{a^2 - x^2}$
  $u = a\sin(\theta),\ du = a\cos(\theta),\ \sqrt{a^2 - x^2} = a\cos(\theta)$
- $\sqrt{a^2 + x^2}$
  $u = a\tan(\theta),\ du = a\sec^2(\theta),\ \sqrt{a^2 + x^2} = a\sec(\theta)$
- $\sqrt{x^2 - a^2}$
  $u = a\sec(\theta),\ du = a\tan(\theta)\sec(\theta),\ \sqrt{x^2 - a^2} = a\tan(\theta)$

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- What's the right substitution ofr $\int \frac{x}{\sqrt{3-2x-x^2}}dx$?
  $x + 1 = 2\sin(\theta)$

Strategy for Integration

- Simplify the Integrand if Possible
- Look for an Obvious Substitution
- Clasify the Integrand according to Its Form
  - Trigonmetric functions
  - Rational functions
  - Integration by parts
  - Radicals
- What shold I do when they don't work?

Strategy for Integration

- **Try Again!!!**
  - Try substitution
  - Try parts
  - Manipulate the integrand
  - Relate the problem to previous problems
  - Use several methods