Laplace Transform Table

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right) \ \ c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$

Formula sheet

Fourier series: For a $2L$-periodic function $f(x)$, the Fourier series for f is

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L},$$

where for $n = 1, 2, \cdots,$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)dx, \quad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx, \quad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx.$$

Heat equation 1: The solution of the heat equation $\alpha^2 u_{xx} = u_t$, $0 < x < L$, $t > 0$, satisfying the (fixed temperature) homogeneous boundary conditions $u(0,t) = u(L,t) = 0$ for $t > 0$ with initial temperature $u(x,0) = f(x)$ has the general form

$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2\alpha^2 t/L^2} \sin\frac{n\pi x}{L}, \quad \text{where} \quad c_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx.$$

Heat equation 2: The solution of the heat equation $\alpha^2 u_{xx} = u_t$, $0 < x < L$, $t > 0$, satisfying the insulated boundary conditions $u_x(0,t) = u_x(L,t) = 0$ for $t > 0$ with initial temperature $u(x,0) = f(x)$ has the general form

$$u(x,t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2\alpha^2 t/L^2} \cos\frac{n\pi x}{L}, \quad \text{where} \quad c_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx.$$

Wave equation: The solution of the wave equation $\alpha^2 u_{xx} = u_{tt}$, $0 < x < L$, $t > 0$, satisfying the homogeneous boundary conditions $u(0,t) = u(L,t) = 0$ for $t > 0$ and initial conditions $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$ for $0 \le x \le L$ has the general form

$$u(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\left(c_n \cos\frac{n\pi\alpha t}{L} + k_n \sin\frac{n\pi\alpha t}{L}\right)$$

where

$$c_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx \quad \text{and} \quad k_n = \frac{2}{n\pi\alpha}\int_0^L g(x)\sin\frac{n\pi x}{L}dx.$$

Laplace equation: The solution of the Laplace equation $u_{xx} + u_{yy} = 0$, $0 < x < a$, $0 \le y \le b$, satisfying the boundary conditions $u(x,0) = u(x,b) = 0$ for $0 < x < a$ and $u(0,y) = 0$ and $u(a,y) = f(y)$ for $0 \le y \le b$ has the general form

$$u(x,y) = \sum_{n=1}^{\infty} c_n \sinh\frac{n\pi x}{b}\sin\frac{n\pi y}{b} \quad \text{where} \quad c_n = \frac{2}{b\sinh(\frac{n\pi a}{b})}\int_0^b f(y)\sin\frac{n\pi y}{b}dy.$$