1. The exam is closed books. No textbooks or other materials can be used
during the exam. No calculators are allowed.
2. You must use a # 2 pencil on the mark-sense sheet.
3. On the mark-sense sheet, fill in the course number which is MA266 and
leave the test/quiz number blank. Fill in your name, the 10-digit Purdue ID
and blacken the appropriate spaces. Sign the mark-sense sheet.
4. Fill in your name and the section number on the question sheets.
5. Turn in both the mark-sense sheets and the question sheets when you are
finished.
6. Show your work on the question sheets. Although no partial credit will be
given, any disputes about grades or grading will be settled by examining your
written work on the question sheets.
Problem 1. Let \( y = y(t) \) be the solution to the initial value problem

\[
\frac{dy}{dt} + 2y = e^t; \quad y(1) = 1.
\]

Find the value of \( y(2) \).

A. \( \frac{1}{4}(e - 1) \);

B. \( \frac{1}{4}(e + 1) \);

C. \( \frac{1}{4}(e^2 - 1) \);

D. \( \frac{1}{4}(e^2 + 1) \);

E. NONE OF THE ABOVE.
Problem 2. Let $y = y(t)$ be the solution to the initial value problem

$$\frac{dy}{dx} = \frac{3x^2 + 4x - 4}{2y - 4}; \quad y(1) = 3.$$ 

Find the value of $y(2)$.

A. 5;  
B. −1;  
C. $2 - \sqrt{10}$;  
D. $2 + \sqrt{10}$;  
E. DO NOT EXIST.
Problem 3. If the following differential equation is exact, select the implicit solution to the initial value problem

\[(e^x \sin y - 2y \sin x) + (e^x \cos y + 2 \cos x + 2y)y' = 0; \quad y(0) = \pi.\]

If it is not exact, select "NOT EXACT".

A. \(e^x \cos y - 2y \sin x = -1;\)

B. \(e^x \sin y + 2y \cos x = 2\pi;\)

C. \(e^x \sin y + 2y \cos x + y^2 = 2\pi + \pi^2;\)

D. \(e^x \sin y - 2y \cos x + y^2 = -2\pi + \pi^2;\)

E. NOT EXACT.
Problem 4. Let $y = y(t)$ be the solution to the initial value problem
\[
\frac{dy}{dx} = \frac{x^2 + y^2}{xy}, \quad x > 0; \quad y(1) = 2.
\]
Find the value of $y(e)$.

A. 0;
B. $\sqrt{6} - e$;
C. $\sqrt{6}$;
D. $e\sqrt{6}$;
E. $6\sqrt{e}$. 

Problem 5. A tank initially contains 40 L of water with 10 g of salt in solution. A mixture containing concentration of 0.5 g/L of salt enters the tank at the rate of 4 L/min and the well-stirred mixture leaves the tank at the rate of 2 L/min. Find the amount of salt in the tank after 20 minutes.

A. 15 g;
B. 25 g;
C. 35 g;
D. 45 g;
E. 55 g.
Problem 6. A ball with mass 1 kg is thrown upward with initial velocity 5 m/s from the roof of a building 20 m high. Assume that the air resistance is of magnitude $v^2/1200$ (in N) directed opposite to the velocity $v$ (in m/s). Let $v(t)$ be the velocity of the ball after $t$ seconds (in m/s). Which of the following initial value problem describes the motion of the ball when it is rising. (Take the gravity constant to be $g = 9.8$ m/s$^2$.)

A. $\frac{dv}{dt} = 5 + 9.8t - \frac{v^2}{1200}, v(0) = 20$
B. $\frac{dv}{dt} = 5 - 9.8t - \frac{v^2}{1200}, v(0) = 20$
C. $\frac{dv}{dt} = 9.8 - \frac{v^2}{1200}, v(0) = 5$
D. $\frac{dv}{dt} = -9.8 - \frac{v^2}{1200}, v(0) = 5$
E. NONE OF THE ABOVE.
Problem 7. How many \textbf{unstable} equilibrium solutions are there for the autonomous equation

\[ \frac{dy}{dt} = y^4 - 5y^2 + 4. \]

A. 0;
B. 1;
C. 2;
D. 3;
E. 4.
Problem 8. Use Euler’s method to find approximate value of $y(0.2)$ of the following initial value problem with step size $h = 0.1$,

$$y' = y^2 - t, \quad y(0) = 1.$$ 

A. 1.1;  
B. 1.211;  
C. 1.22;  
D. 1.221;  
E. 2.21.
Problem 9. Which of the following is a fundamental set of solution of

\[ y'' - 4y' + 5y = 0. \]

A. \( \{e^{2t}, e^{-2t}\} \);  
B. \( \{e^{2t} \cos t, e^{2t} \sin t\} \);  
C. \( \{e^{-2t} \cos t, e^{-2t} \sin t\} \);  
D. \( \{e^t \cos(2t), e^t \sin(2t)\} \);  
E. \( \{e^{2t}, te^{2t}\} \).
Problem 10. Let $y = y(t)$ be the solution of the initial value problem

$$4y'' + 4y' + y = 0, \quad y(0) = 1, \quad y'(0) = \frac{1}{2}.$$ 

Find the coordinates of the maximum point of $y = y(t)$.

A. $(-1, 0)$;
B. $(0, 1)$;
C. $(1, \frac{2}{\sqrt{e}})$;
D. $(3, -2\sqrt{e^3})$;
E. DO NOT EXIST.