questions Review

1. The matrix below represents the augmented matrix of a system of linear equations. Assume that the variables in this system are x_1, x_2, x_3, x_4, x_5 , and x_6 , and let A be the coefficient matrix:

$$\begin{pmatrix} 1 & 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 & c \\ 0 & 0 & 0 & 0 & 0 & 1 & d \end{pmatrix} \qquad \begin{array}{c} A & 4 \times 6 \\ = & \text{cholor form} \\ \text{not reduced echelor} \\ \text{form}. \end{array}$$

Ray (A) = 4

[2aux (A)b)=4

 $|nu||_1 h_y(h) = 2$ = 6-4

why?

are in 12 4

Which of the following statements are **true**?

(i) For any given c and d, the system above is consistent.

(ii) The coefficient matrix A is in reduced echelon form.

(iii) The right hand side vector is in the column space of matrix A.

(iv) The system has no solution.

e her yariables (v) The system has infinitely many solutions.

A. (i), (ii) only

B. (i), (ii), (iii) only

C. (ii), (iii), (iv) only

D. (i), (iii), (v) only

E. all of the above

$$A \times = P \iff \begin{cases} 3^{m} & x \\ y & y \\ y & y \end{cases} = \begin{cases} y^{m} & y \\ y & y \\ y & y \end{cases} = \begin{cases} y^{m} & y \\ y & y \\ y & y \end{cases} = P$$

consistent (=>b in Span Lat, ... and (has solutions)

ā, .., ān are linearly moll pendent (x, ā, a...+ Xn ān = 6 (=) x,= ...= x=0)

Rank (A) + Nullity (A) = m = nr o)

If xo solution of Ax = b that all vectors in Ax = b that all vectors in Ax = bXo+ Mul(A) are solu

3. Which of the following statements is **false**?

Folde

- **A.** If $\{v_1, v_2, v_3\}$ is linearly dependent, then v_3 is a linear combination of v_1 and v_2 .
- **B.** Suppose that the columns of A are v_1, v_2 , and v_3 . Then the matrix equation $A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = b$ is equivalent to the vector equation $x_1v_1 + x_2v_2 + x_3v_3 = b$.
- C. Suppose that X_0 is a solution to the linear system AX = b. Then $\{X | AX = b\} = X_0 + \{X | AX = 0\}$. (A)
- **D.** The columns of A are linearly independent if and only if A has a pivot position in every column.
- **E.** A homogeneous linear system has a non-trivial solution if and only if it has at least one free variable.

 $\exists x: \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \end{bmatrix} \right\}$

- 8. Let A be an $m \times n$ matrix. Which of the following statement is necessarily **true**?
 - **A.** The nullity of A is the same as the nullity of A^T .
 - **B.** The rank of A is the same as the rank of A^T .
 - C. The column space of A is the same as the null space of A^T .
 - **D.** The columns of A form a basis of the column space of A.
 - **E.** The columns of A^T form a basis of the null space of A.

Part(A) + Nullity (A) =
$$n$$

Part(AT) + Nullity (AT) = m

AT $n \times m$

Fact: Part(A) = Part(AT) < $min \{ m, n \}$

Col(A) $\subseteq IR^m$

dim (Col(A)) $\subseteq m$

Part $(A \cap A) \subseteq m$

And $(A \cap A$

18. Which of the following subsets of the vector space
$$\mathbb{R}^3$$
 are subspaces of \mathbb{R}^3 ?

(i) The set of all vectors $v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ with the property $2xyz = 0$.

(ii) The set of all the solutions of the equation $x - 5y + 2z = 0$.

(iii) The set of all solutions for the system $\begin{bmatrix} 2 & 3 & 2 \\ 5 & 2 & 8 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

(iv) The set of all the solutions of the equation $x + 3y = 2z + 1$.

A. (ii) and (iii) only

B. (ii) and (iv) only

C. (iii) and (iv) only

D. (ii), (iii) and (iv) only

 \mathbf{E} . (i), (ii), (iii) and (iv)

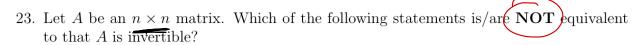
Rewark The solution set of any homogeneous exitem

A M XN

A X = 0 IS a subspace of Rⁿ

A MAI(A)

in not a subspace $A \times = 0$ $A \times = 0$



- (i) Columns of A are linearly independent.

 A inverhble
- (ii) A is diagonalizable.
- (iii) Columns of A is an orthonormal set.

The linear system AX = b always has solution for any $b \in \mathbb{R}^n$.

Cank (1-1 = N - null | M / 1-1 | M (iv) The dimension of the null space of A is 0.

dim (G) (H) = M

Rank (H) = M

Some REF = [1 1 1 2 1]

- **A.** (i), (ii) and (iii) only.
- **B.** (i) and (ii) only.
- C. (ii) and (iii) only.
- **D.** (i) and (iv) only.
- **E.** (ii), (iv), (v) only.

Recall A mxn

A invertible (=) RREF(A) = In

Olim(Col(A))=M (=) RAUK(A)=M

- END OF CLASS