Functional Analysis, Spring 2020
Homework #9
This assignment is due on Wednesday, April 1

1. Let H be a separable Hilbert space and assume that $T \in L(H)$ is a normal operator. Suppose that the spectrum of T is not connected. Show that there is a nonzero selfadjoint projection $p = p^2 = p^*$ in $C^*\{1, T\}$ such that $Tp = pT$.

2. Let H be a separable Hilbert space and assume that $T \in L(H)$ is a normal operator. Show that any isolated point λ in the spectrum of T must be an eigenvalue. In other words, there is a nonzero vector $x \in H$ such that $Tx = \lambda x$.

3. Let H be a separable Hilbert space. Let $S, T \in L(H)$ be such that $\|Sx\| \leq \|Tx\|$ for all $x \in H$. Show that there is $V \in L(H)$ such that $S = VT$.