Lesson 1: Review of Functions

- Definition
- Domain and Range
- Composition of functions
- Symmetry
- Transformation

What is a function?

Rule that assigns exactly one output for each input

output

$$y = f(x)$$

input

eg: $f(x) = x^3 - 5x + 1$

input = 1 ⇝ output = $f(1) = 1^3 - 5(1) + 1 = -3$

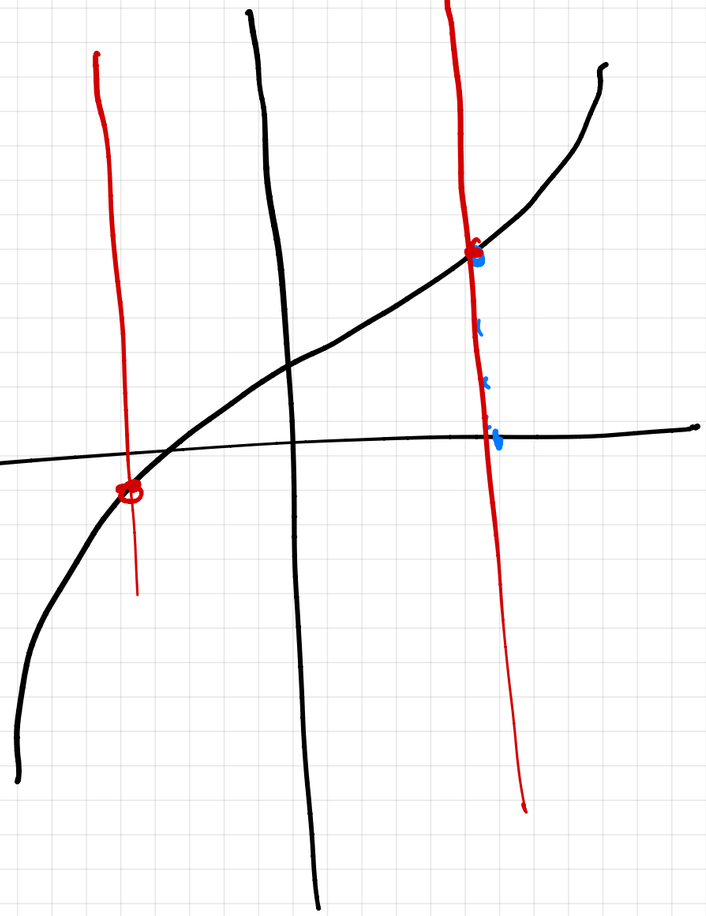
input = 0 ⇝ output = $f(0) = 0 - 0 + 1 = 1$

input = a ⇝ output = $f(a) = a^3 - 5a + 1$

Vertical Line Test

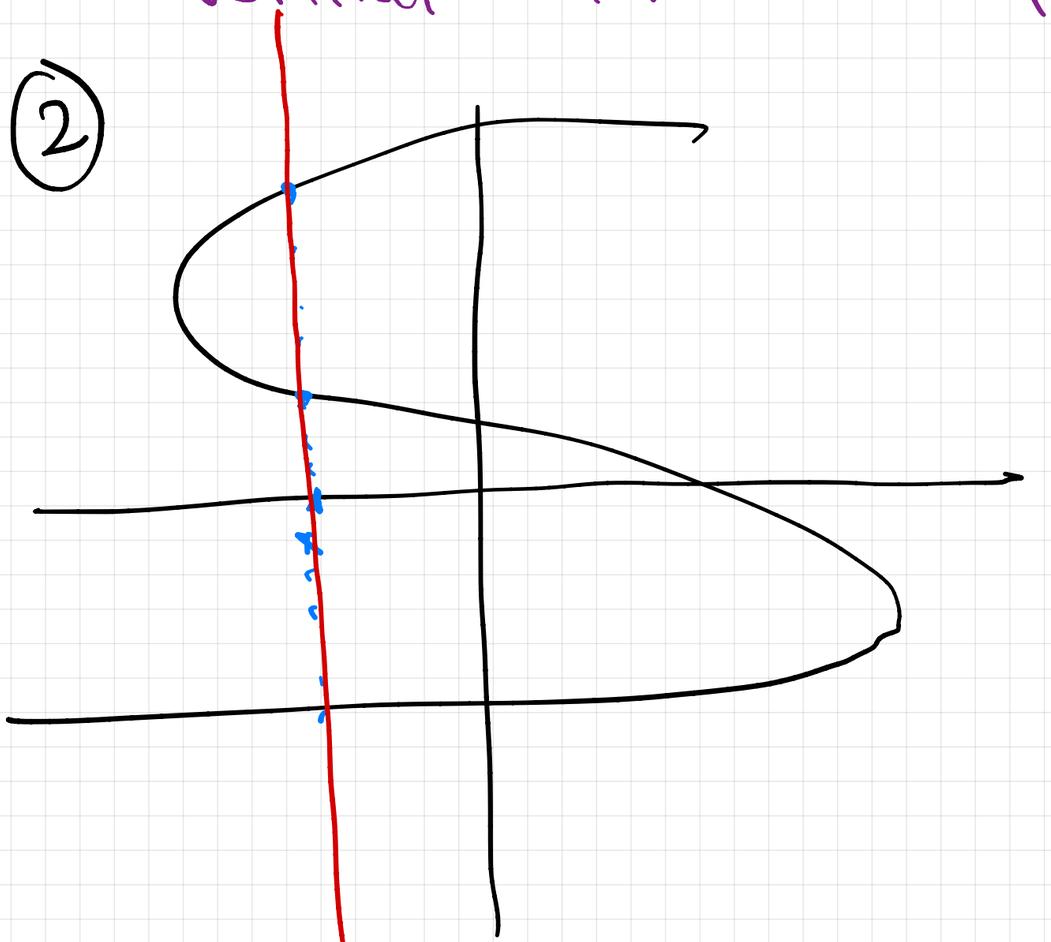
on a graph: horizontal axis → inputs
vertical axis → outputs

①



function

②



NOT function.

Domain and Range

Domain: all acceptable inputs

Range: all possible outputs

Eg: $f(x) = \sqrt{x+5}$

input = 4 ⇝ output = $\sqrt{9} = 3$

input = -1 ⇝ output = 2

input = -4 ⇝ output = 1

input = -5 ⇝ output = 0

input = -6 ⇝ Not acceptable

Acceptable when $x+5 \geq 0$

$x \geq -5$

D: $[-5, \infty)$, R: $[0, \infty)$

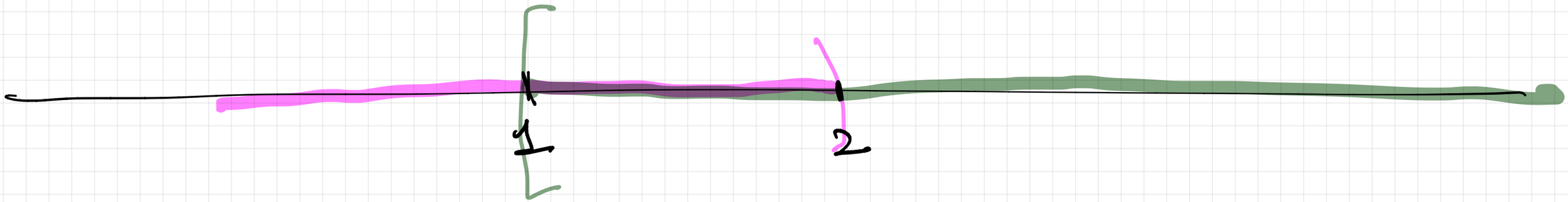
eg: Find domain of $f(x) = \sqrt{x-1} - \frac{1}{\sqrt{2-x}}$

acceptable when $x-1 \geq 0$

$x \geq 1$

acceptable when $2-x > 0$

$x < 2$



Domain: $[1, 2)$.

Composition of functions

$f(x) = \frac{1}{x}$, $g(x) = \sqrt{2-x}$

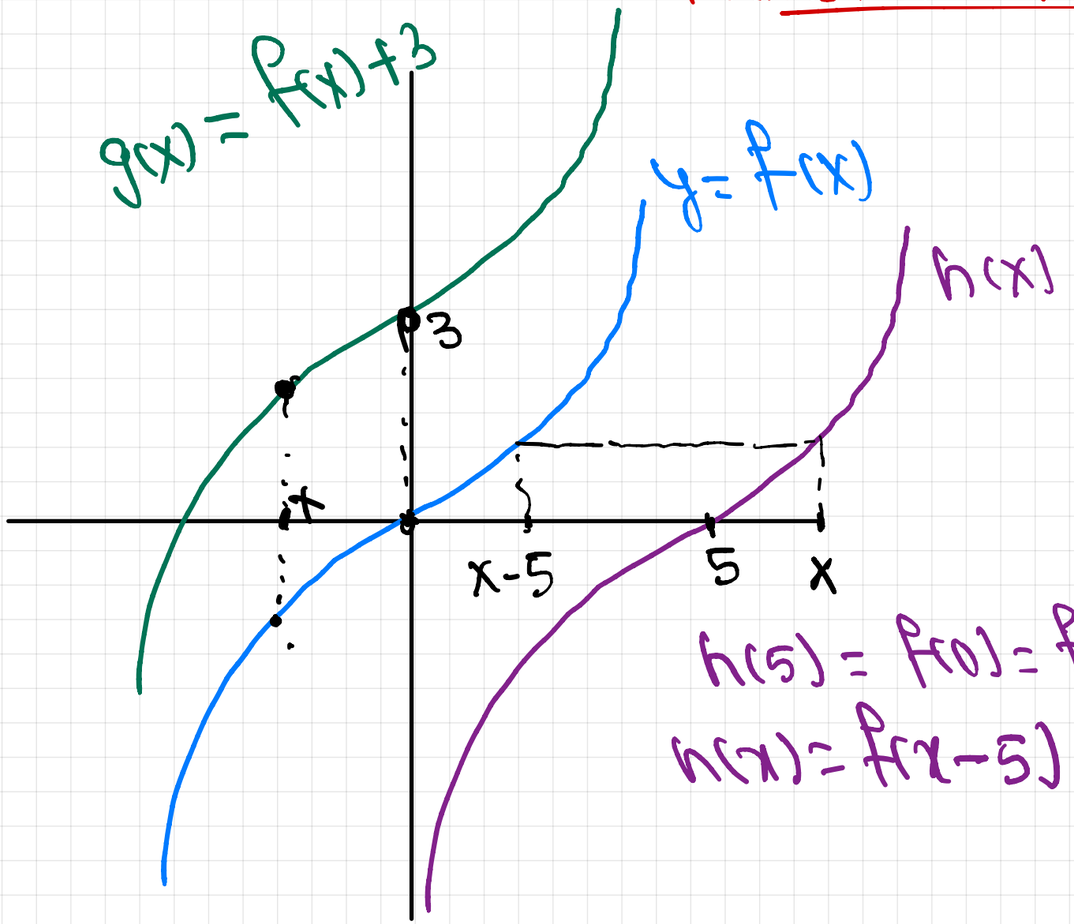
$f(g(x)) = \frac{1}{g(x)} = \frac{1}{\sqrt{2-x}}$

$g(f(x)) = \sqrt{2-f(x)} = \sqrt{2-\frac{1}{x}}$

$f(g(x))$ need not be equal to $g(f(x))$.

Symmetry

Transformations: Shift, Stretch, Reflection.



$h(5) = f(0) = f(5-5)$

$h(x) = f(x-5)$

Vertical shift:

$y = f(x) + k$

$k > 0$ ⇝ shift up

$k < 0$ ⇝ shift down.

Horizontal shift:

$y = f(x-k)$

$k > 0$ ⇝ shift Right

$k < 0$ ⇝ Shift Left

Vertical stretch:

$y = kf(x)$

$k > 1$ ⇝ stretch

$0 < k < 1$ ⇝ compress

Horizontal stretch:

$y = f(kx)$

$0 < k < 1$ ⇝ stretch

$k > 1$ ⇝ Compression.

Reflect about y-axis: (Horizontal Reflection)

$y = f(-x)$

about x-axis: $y = -f(x)$

$f(-x) = f(x)$ } EVEN fn eg: x^2

$f(-x) = -f(x)$ } ODD fn eg: x^3 .

$y = f(x)$

$g(x) = f(-x)$

$f(x)$

$-x$

x

$h(x) = -f(x)$

$-f(x)$

Sketch the graph of $f(x) = 6(x+5)^2 + 2$ with the help of $y = x^2$