MA 161 - Lesson 2 (1.3)

Today: Inverse of Function

One to One Function

Horizontal Line Test

Exponential and Logarithmic Functions

Properties of Logarithmic functions

Office Hours: Monday, Wednesday, Friday 2:45PM - 4:15PM, MATH 842

Announcements: Enroll in My Lab Math (MLM) as soon as possible

Review Syllabus and Schedule

For faster response to emails: Copy your TA and write your section number

Pre-Notes

Additional Resource: Pre-recorded videos

Warmup: output $\qquad$ input

$$y = f(x) = x^3$$

find the value of input when output = 8

find x $\qquad$ given $y = 8$

$$8 = x^3 \rightsquigarrow x = 8^{1/3} = 2$$

input: 27 $\rightsquigarrow$ output = 3

input = y $\rightsquigarrow$ output = $x = y^{1/3}$

$= x^3$

New function! $g(x) = x^{1/3}$ is the inverse function for the given $f(x) = x^3$

Notation: $g(x) = f^{-1}(x)$

eg: $f(x) = x^2$

find input when output $= 4$

can be 2 or -2

Let output $= y \rightsquigarrow$ input $= \sqrt{y}$ or $-\sqrt{y}$

inverse does not exist for $f(x) = x^2$

Q: When does inverse exist?

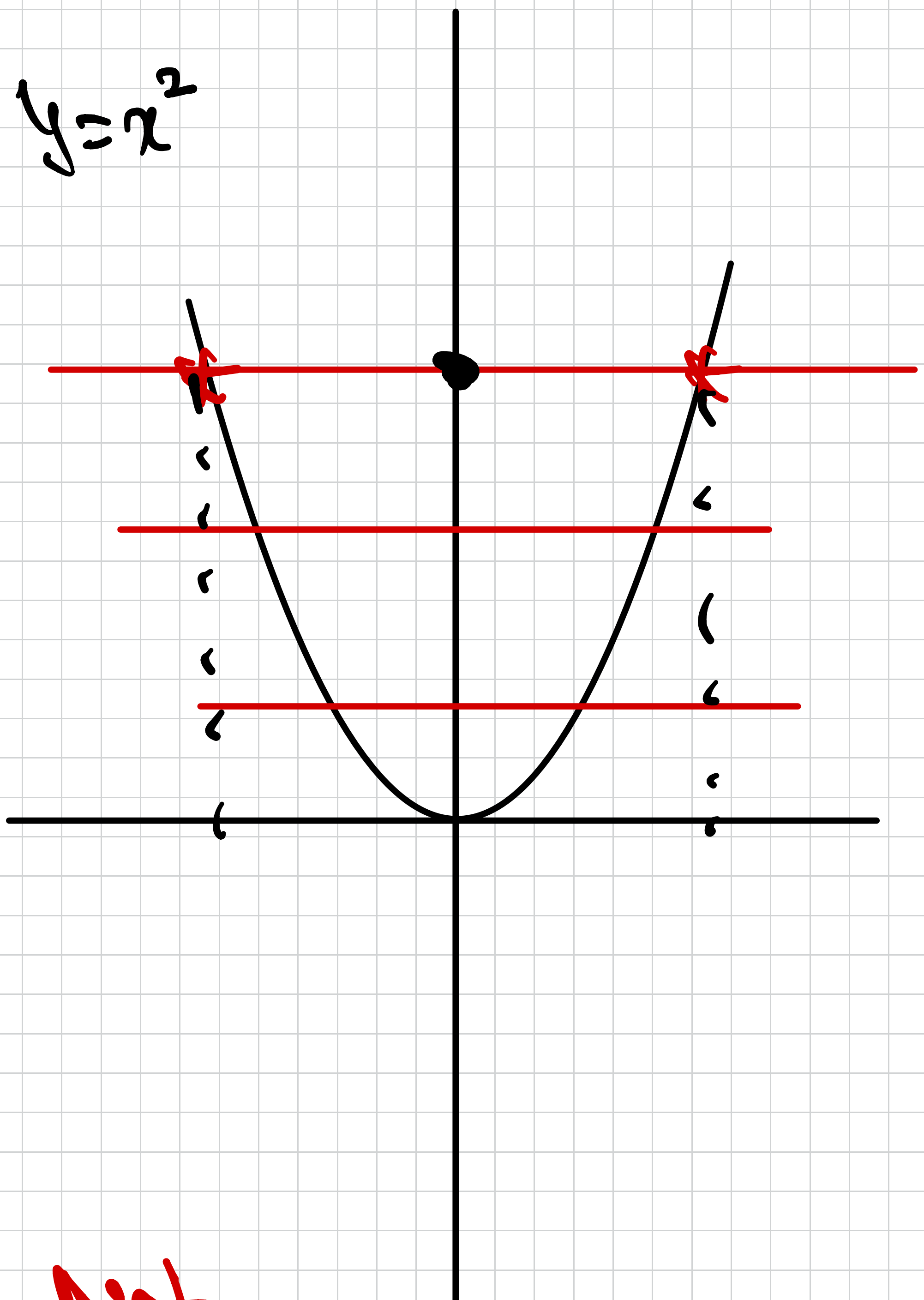
When the function is 1-1 in its domain

No two inputs have the same output.

Horizontal Line Test

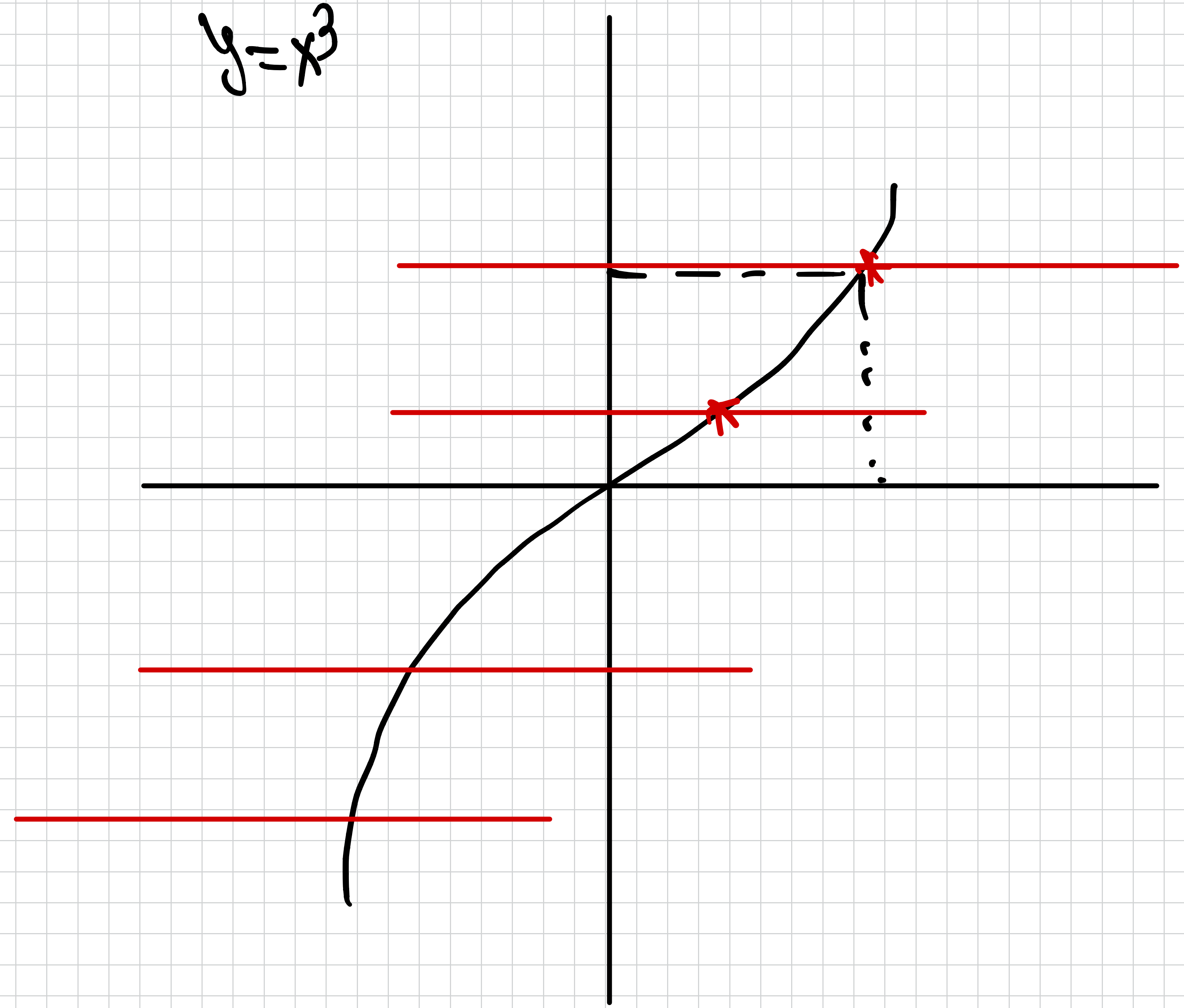
used to determine whether the Given Graph is 1-1 or not

$y=x^2$



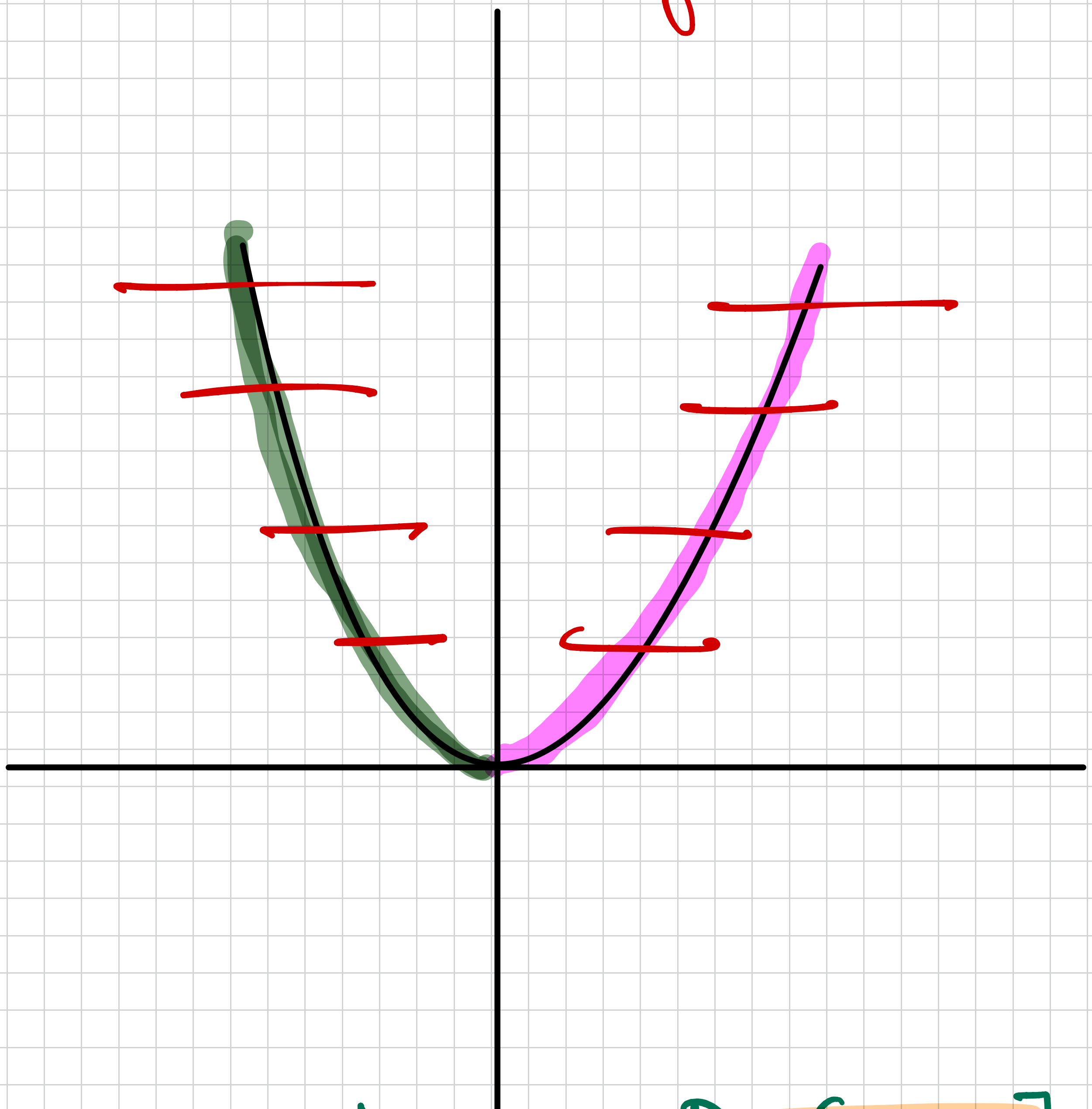
Not 1-1

$y=x^3$



1-1

Restricting Domain to make a function 1-1



① Restrict Domain to $[0, \infty)$

Range = $[0, \infty)$

x^2 is invertible in this Domain

y is output $\rightsquigarrow$ input $= \sqrt{y}$

$f^{-1}(x) = \sqrt{x}$ is the inverse function

D: $[0, \infty)$

R: $[0, \infty)$

② Restrict D: $(-\infty, 0]$, R: $[0, \infty)$

Output $= 4 \rightsquigarrow$ input $= -2$, Output $= 100 \rightsquigarrow$ input $= -10$

Output $= y \rightsquigarrow$ input $= -\sqrt{y}$

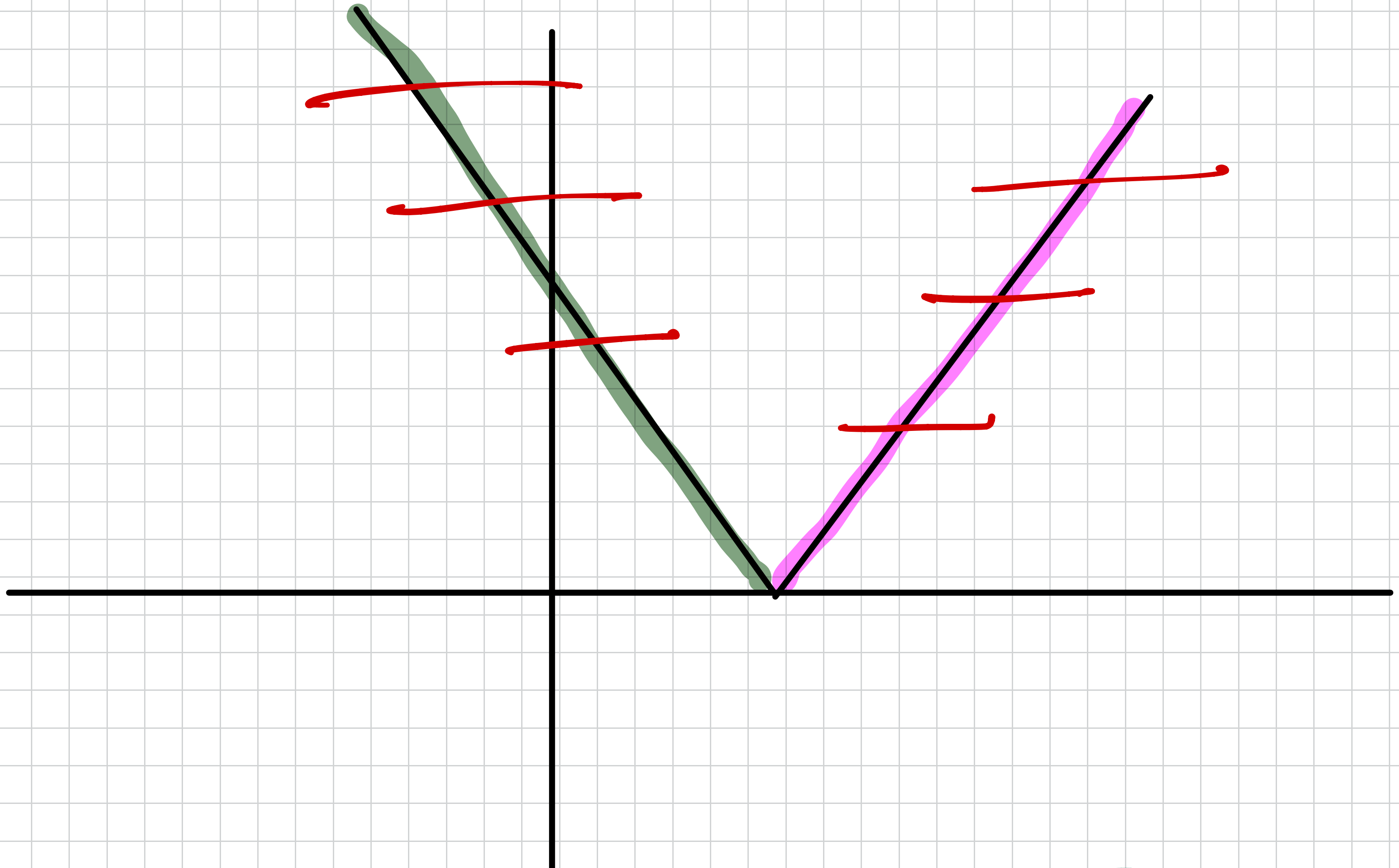
$f^{-1}(x) = -\sqrt{x}$, D: $[0, \infty)$, R: $(-\infty, 0]$

eg: $f(x) = |x-5|$

$|-8| = 8 = -(-8)$

$|8| = 8$

$$f(x) = \begin{cases} x-5 & x-5 \geq 0 \rightsquigarrow x \geq 5 \\ -(x-5) = 5-x & \underbrace{x-5<0}_{x<5} \end{cases}$$



Not invertible on $(-\infty, \infty)$

① Restrict: D: $[5, \infty)$
R: $[0, \infty)$

$y = x-5 \rightsquigarrow x = y+5$

$f^{-1}(y) = y+5$ (OR) $f^{-1}(x) = x+5$
D: $[0, \infty)$
R: $[5, \infty)$

② Restrict D: $(-\infty, 5]$, R: $[0, \infty)$

$y = 5-x \rightsquigarrow x = 5-y$

$f^{-1}(y) = 5-y$ (OR) $f^{-1}(x) = 5-x \rightsquigarrow$ D: $[0, \infty)$
R: $(-\infty, 5]$

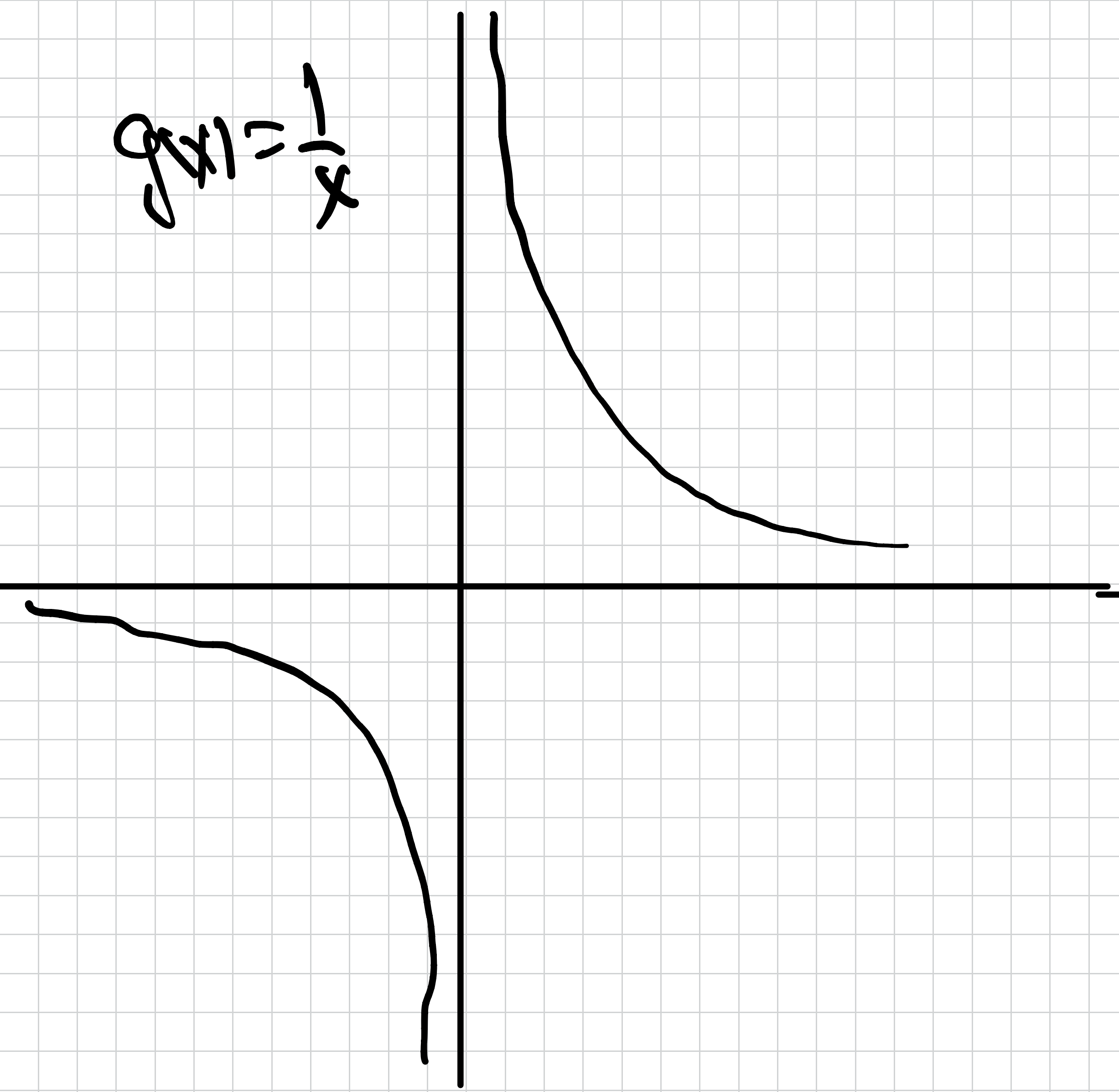
eg: $f(x) = \dfrac{1}{x-3}$, is $f(x)$ 1-1, if so find $f^{-1}(x)$.

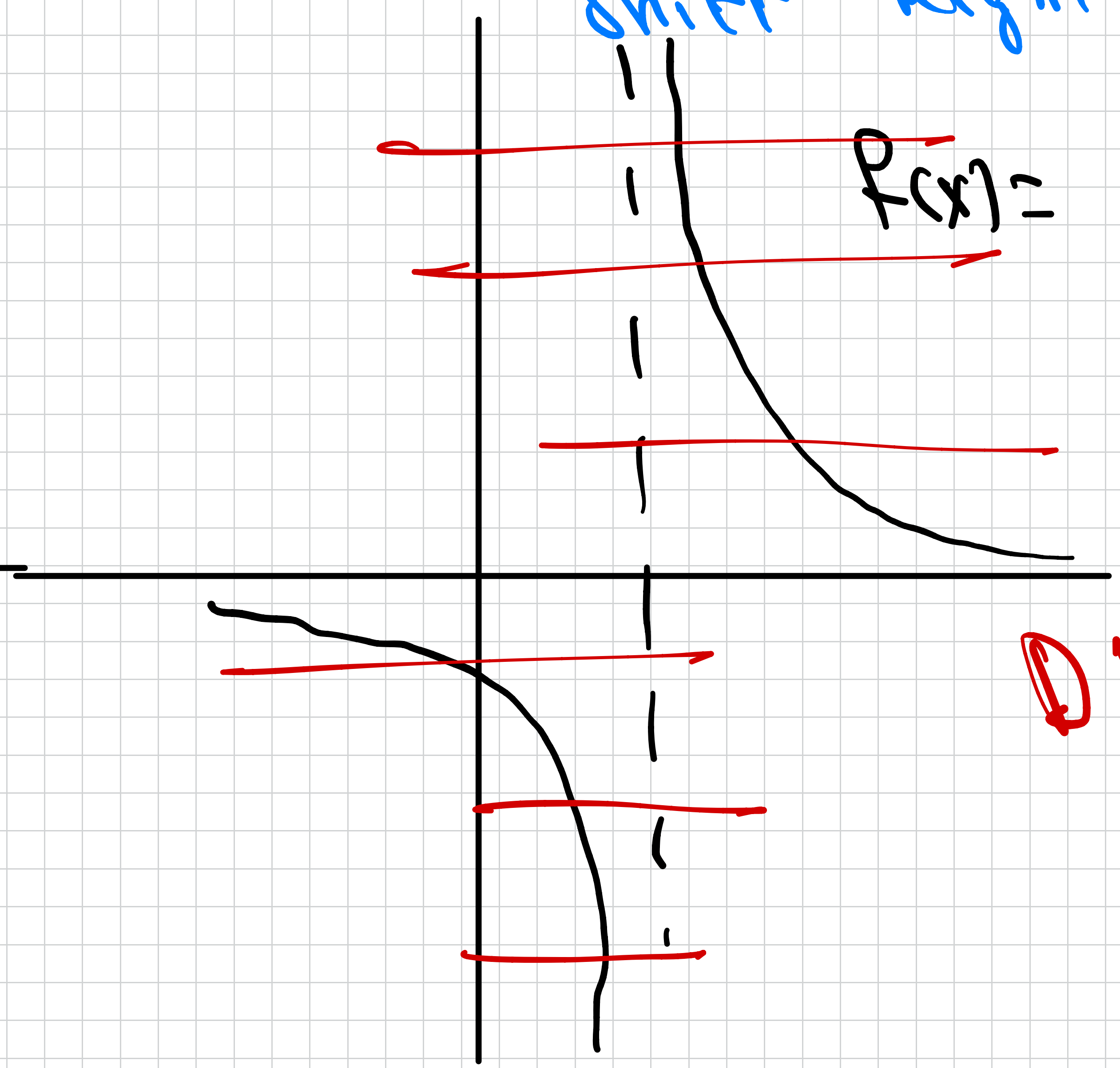
How is this Related to $g(x) = \dfrac{1}{x}$.

$g(x-3) = \dfrac{1}{x-3}$

Shift Right by 3 units

$g(x) = \dfrac{1}{x}$

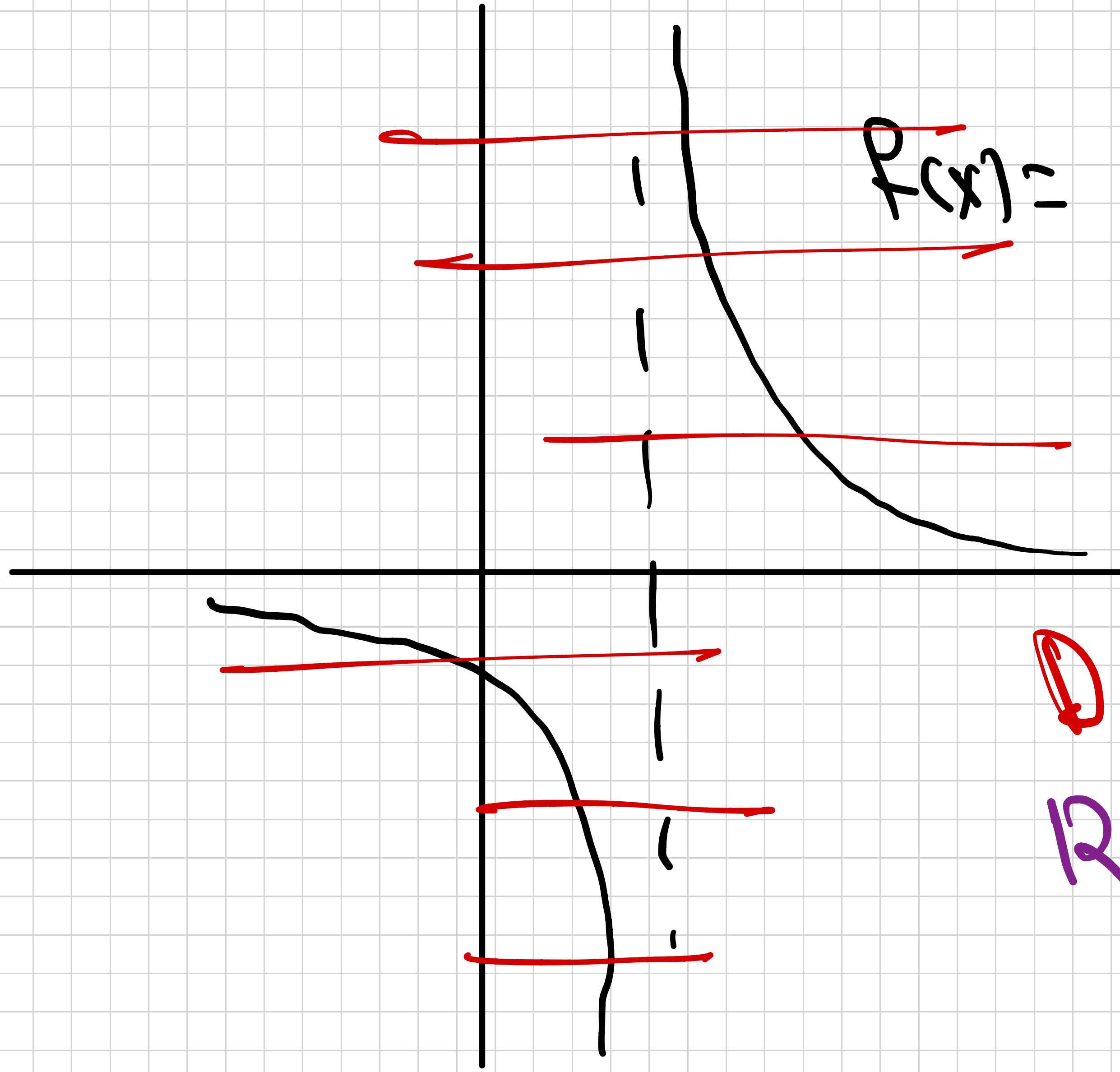




$f(x) = \dfrac{1}{x-3}$

1-1 in the

$D: (-\infty, 3) \cup (3, \infty)$



$f(x) = \frac{1}{x-3}$

1-1 in the

D: $(-\infty,3)\cup(3,\infty)$

R: $(-\infty,0)\cup(0,\infty)$

Finding inverse!

$$y = \frac{1}{x-3}$$

$$x-3 = \frac{1}{y} \rightsquigarrow x = \frac{1}{y}+3$$

$$f^{-1}(y) = \frac{1}{y}+3 \quad \text{or} \quad f^{-1}(x) = \frac{1}{x}+3 \rightsquigarrow$$

D: $(-\infty,0)\cup(0,\infty)$

R: $(-\infty,3)\cup(3,\infty)$

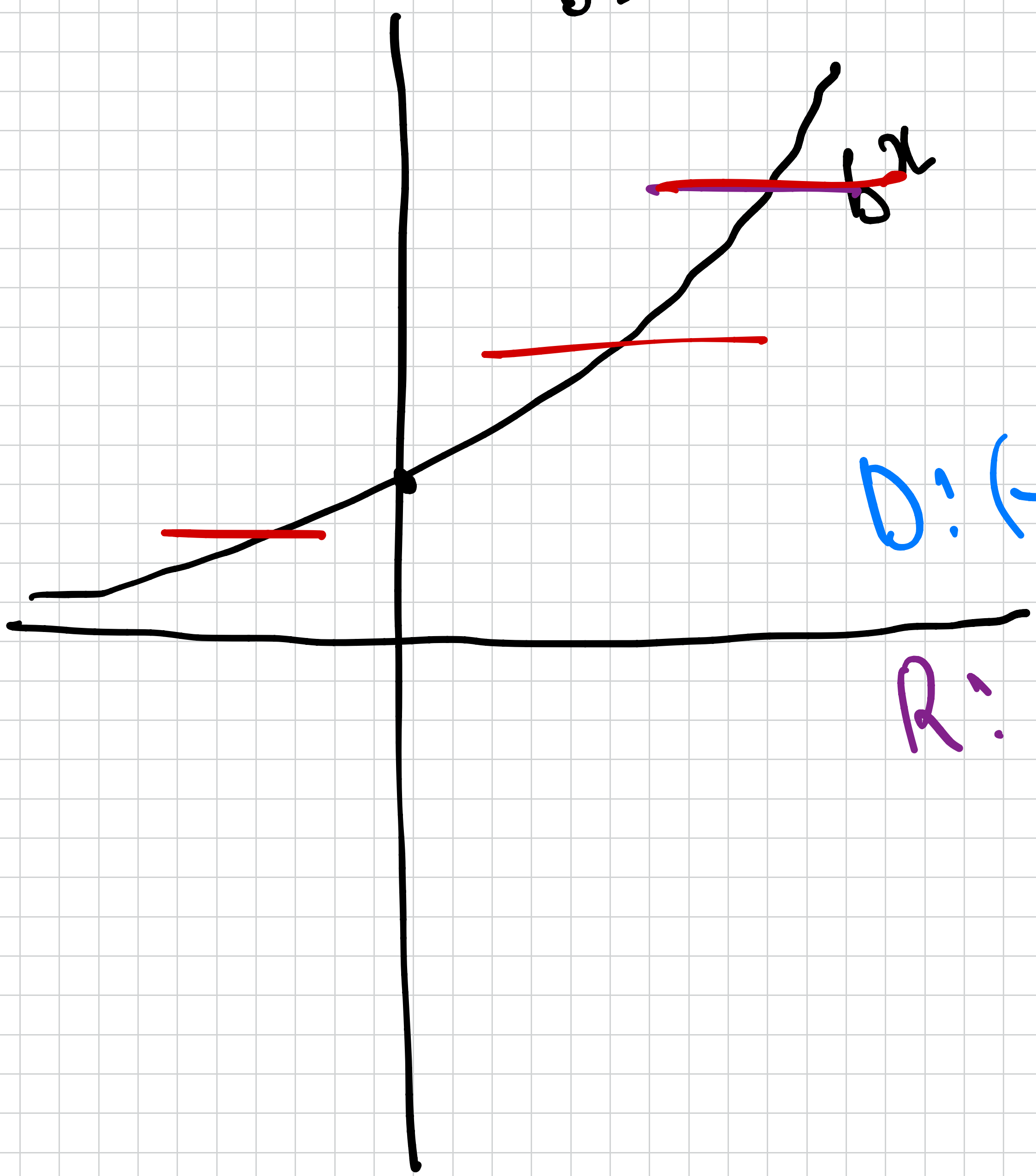
Exponential and Logarithmic Functions

$y = b^x$ ⇝ inverse of $y = f(x) = b^x$

$b > 0$

is Logarithmic function

$f^{-1}(x) = \log_b x$

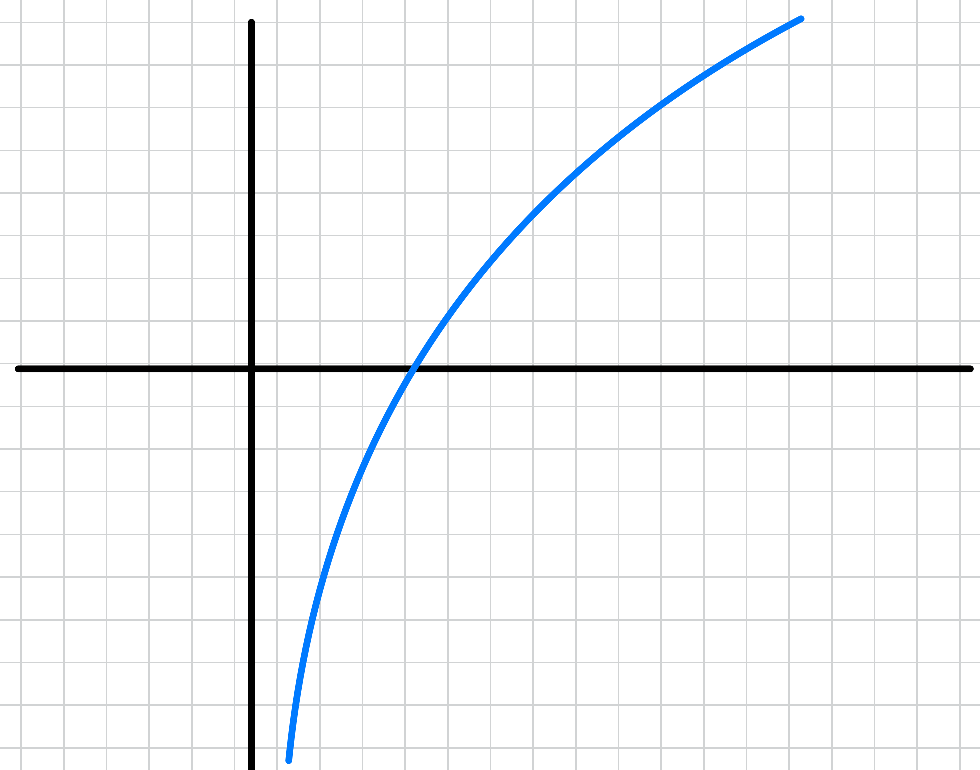


D: $(-\infty, \infty)$

R: $(0, \infty)$

D: $(0, \infty)$

R: $(-\infty, \infty)$



Properties of Logarithm

* $b^0 = 1 \rightsquigarrow \log_b 1 = 0$

  $b^1 = b \rightsquigarrow \log_b b = 1$

$f(f^{-1}(x)) = x$ $\quad$ $f^{-1}(f(x)) = x$

$f(x) = b^x$, $f^{-1}(x) = \log_b x$ $\hookrightarrow$ $b^{\log_b x} = x$

$\log_b b^x = x$

* $\log_b (xy) = \log_b x + \log_b y$

* $\log_b (x/y) = \log_b x - \log_b y$

* $\log_b x^r = r \log_b x$

eg #: find x such that $3^{7x-2} = 18$

take $\log_b$ on both sides

$$\log_3 3^{7x-2} = \log_3 18$$

$$(7x-2)\underbrace{\log_3 3}_{1} = \underbrace{\log_3 3^2}_{2} + \log_3 2$$

$$7x-2 = 2 + \log_3 2$$

$$x = \frac{1}{7}\left(4 + \log_3 2\right)$$

e.g: Write 6^{4x} as exponential base e.

$$e^{\log_e y} = y \quad \text{for any } y$$

Let $y = 6^x$

$$e^{\log_e 6^x} = 6^x$$