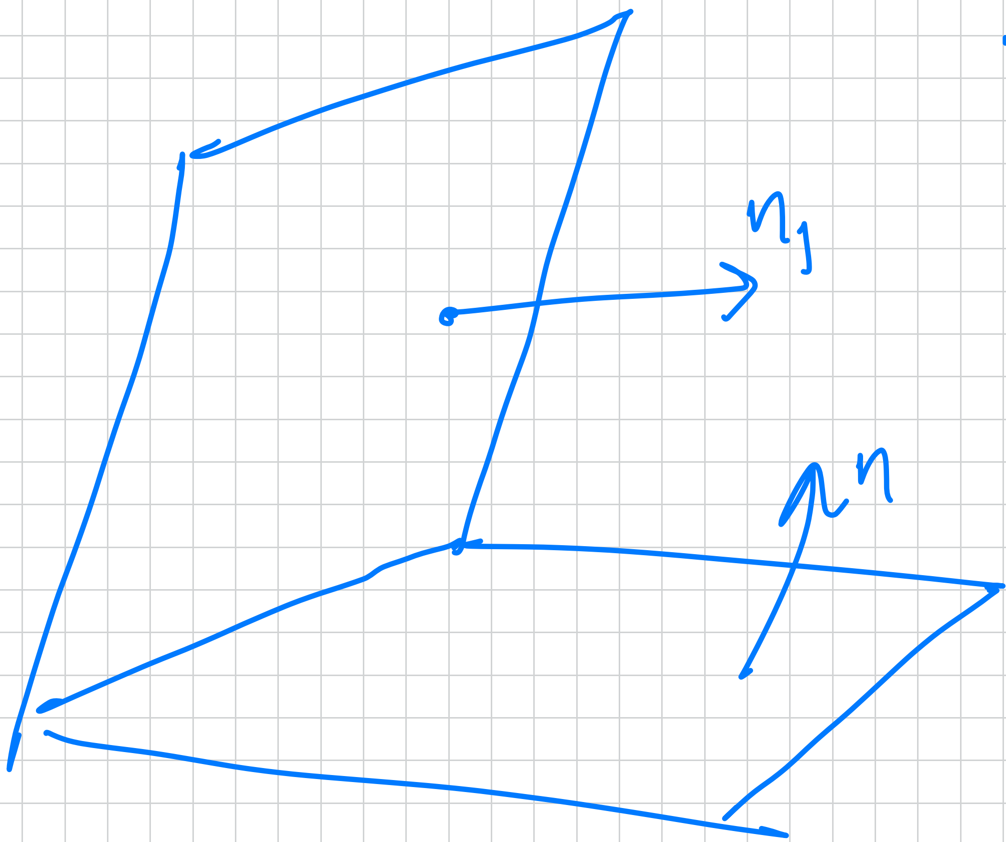
Exam 1 Review

Find equation of plane through $(0,1,2)$ and perpendicular to $\underbrace{x-y+2z=1}_{}$ and $\underbrace{3x+2z=-4}_{}$

$n_1 = \langle 1,-1,2 \rangle$ $\qquad$ $n_2 = \langle 3,0,2 \rangle$



two planes are $\perp$ then normal vectors are $\perp$

$\vec{n}$ is what we want $\rightarrow$ $\perp$ to both n_1, n_2

$$\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 3 & 0 & 2 \end{vmatrix} = \boxed{\langle -2, 4, 3 \rangle}$$

Plane through $(0,1,2)$ with $\vec{n} = \langle -2, 4, 3 \rangle$

$$-2(x-0)+4(y-1)+3(z-2)=0.$$

$$-2x+4y+3z-10=0.$$

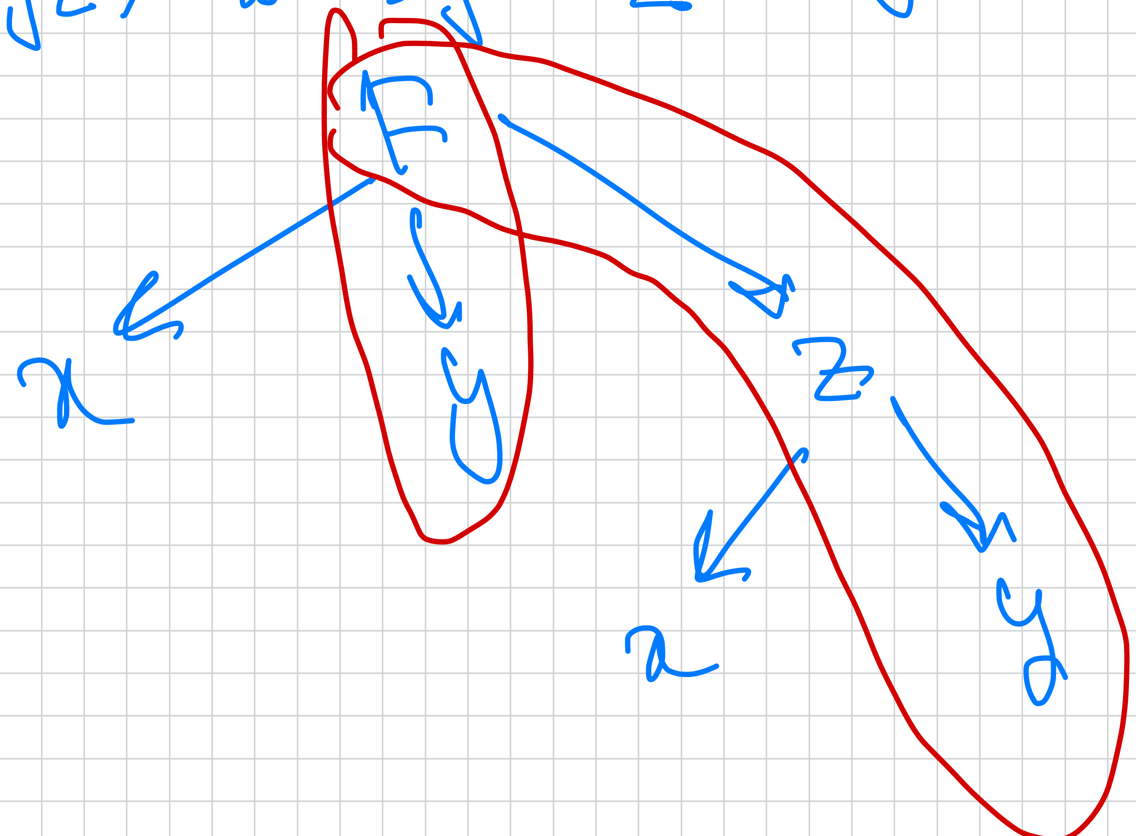
z is implicitly given as function of x, y

$\cos(xyz) = x + 3y + 2z$, evaluate $\dfrac{\partial z}{\partial y}(0,1)$ $\rightarrow$ $x = 0$, $y = 1$

$$F(x,y,z) = \cos(xyz) - x - 3y - 2z = 0$$

$\cos(0) = 0 + 3 + 2z$

$z = -1$



$$F_y + F_z \cdot \frac{\partial z}{\partial y} = 0$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{\left(-\sin(xyz)\cdot xz - 3\right)}{-\sin(xyz)\,xy - 2}$$

$$\frac{\partial z}{\partial y}(0,1) = -\frac{(-3)}{-2} = -\frac{3}{2}.$$

$f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$

I $\quad f_x = 2x + y + 3 = 0 \longrightarrow y = -3 - 2x$

II $\quad f_y = x + 2y - 3 = 0$

$\downarrow$ in II

$x + 2(-3 - 2x) - 3 = 0$

$x - 6 - 4x - 3 = 0$

$-3x - 9 = 0 \leadsto x = -3,$

$\downarrow$ in I

$y = -3 - 2(-3)$
$= 3$

$(-3, 3)$ is critical.

$f_{xx} = 2 > 0$

$f_{yy} = 2$

$f_{xy} = 1$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = 3 > 0$

Local Min

$f(x,y) = xy^4 - x - \frac{1}{2}x^2$ on $\mathbb{R}^2$, find Local max/min.

(I) $f_x = y^4 - 1 - x = 0$

(II) $f_y = 4xy^3 = 0 \longrightarrow x = 0$ or $y = 0$

$x = 0$ in (I): $y^4 - 1 = 0$

$y = 1$ or -1

$(0,1)$, $(0,-1)$

$y = 0$ in (I): $-1 - x = 0$

$x = -1$

$(-1, 0)$

$f_{xx} = -1$

$f_{yy} = 12xy^2$

$f_{xy} = 4y^3$

$D = -12xy^2 - (4y^3)^2 = -12xy^2 - 16y^6$

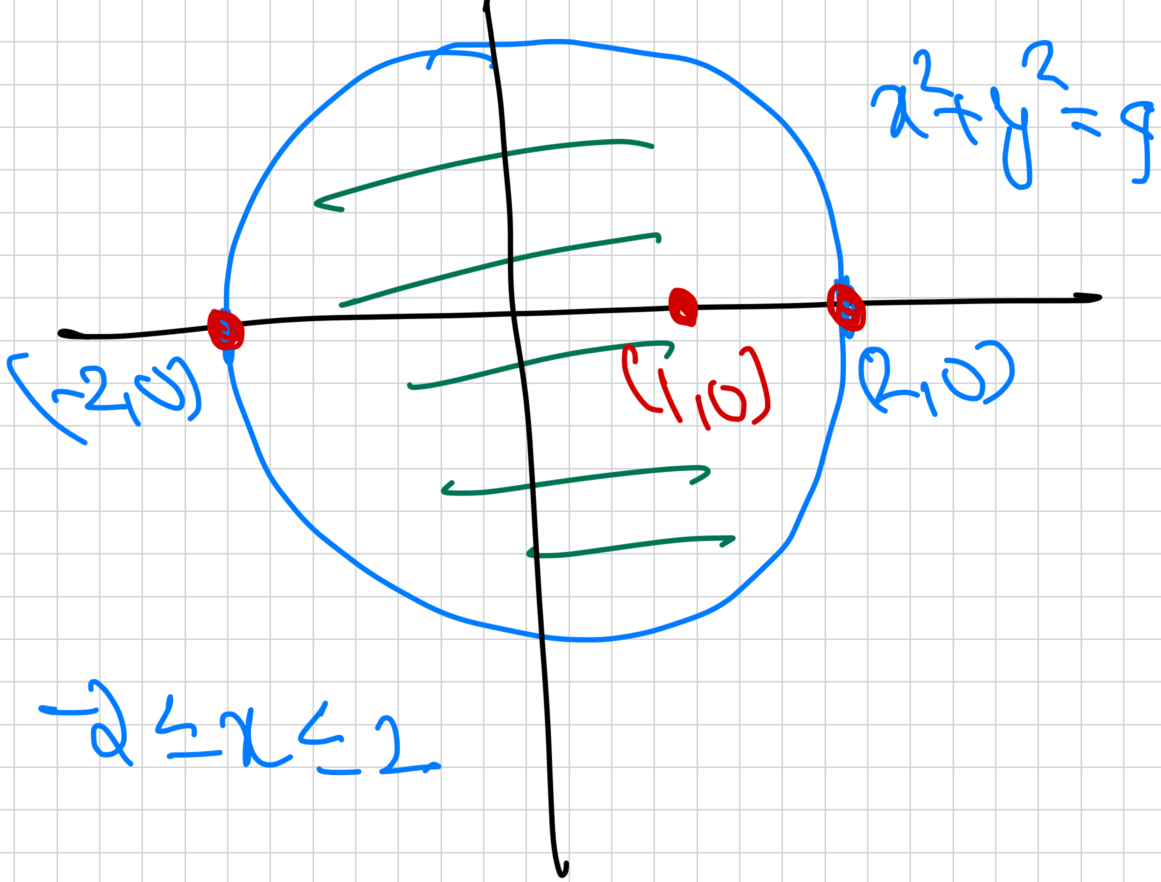
@ $(0,1) \rightsquigarrow D = -16 < 0$

$(0,-1) \rightsquigarrow D = -16 < 0$

$\Big\}$ $(0,1)$, $(0,-1)$ are Saddle

$(-1,0) \rightsquigarrow D = 0$ $\rightsquigarrow$ $(-1,0)$ we cannot conclude.

$f(x,y) = x^2 - 2x + y^2 + 4$



inside:

$f_x = 2x - 2 = 0 \rightsquigarrow x = 1$

$f_y = 2y = 0 \rightsquigarrow y = 0$

On Boundary:

$y^2 = 4 - x^2 \geq 0 \rightsquigarrow -2 \leq x \leq 2$

{in $f(x,y)$)

$x^2 - 2x + (4 - x^2) + 4$

$g(x) = 8 - 2x$, on $[-2,2]$

$x = 2$ or $x = -2$

$\downarrow$ $\downarrow$

$y = 0$ $y = 0$

$(2,0)$ $(-2,0)$

$f(1,0) = 3 \}\, m$

$f(2,0) = 4$

$f(-2,0) = 12 \}\, M$

$\rightsquigarrow M + m = 15.$

Find a such that $(8,a,1)$ is on the tangent plane to $z^2 = e^{xy} - 4x^2y + 3y^2$ at $\boxed{(0,1,2)}$?

Given $F(x,y,z)=0 \rightsquigarrow \vec{\nabla} F$ is the normal vector to tangent plane

$$F(x,y,z) = e^{xy} - 4x^2y + 3y^2 - z^2 = 0$$

$$\vec{\nabla} F = \langle y e^{xy} - 8xy,\ x e^{xy} - 4x^2 + 6y,\ -2z \rangle$$

$$\vec{n} = \vec{\nabla} F(0,1,2) = \boxed{\langle 1, 6, -4 \rangle}$$

Eq. plane through $(0,1,2)$, $\vec{n} = \langle 1, 6, -4 \rangle$

$$1(x-0) + 6(y-1) - 4(z-2) = 0$$

$$x + 6y - 4z + 2 = 0$$

$(8,a,1)$ is on the plane $\rightsquigarrow 8 + 6a - 4 + 2 = 0$

$$6 + 6a = 0$$

$$\Rightarrow \boxed{a = -1}$$

$\vec{r}(t) = \langle 3\sin(2t),\ 4,\ 3\cos(2t) \rangle$

* find arc length from $t=0$ to $t=\pi/3$
* find arc length parametrization starting at $t=0$

$$L = \int_a^b |r'(u)|\, du \quad , \qquad r'(u) = \langle 6\cos(2u),\ 0,\ -6\sin(2u) \rangle$$

$$|r'(u)| = 6$$

$$L = \int_0^{\pi/3} 6\, du = \frac{6\pi}{3}$$

Arc length function: $\quad s(t) = \int_0^t |r'(u)|\, du = 6t$

$$s = 6t$$

$$t = \frac{s}{6}$$

Arc length parametrization. Replace $t = \frac{s}{6}$

$$\left\langle 3\sin\left(\frac{s}{3}\right),\ 4\ ,\ 3\cos\left(\frac{s}{3}\right) \right\rangle$$

$$f(x,y) = \int_{7}^{\sin(xy)} \ln(t)\, dt \quad , \quad \text{find } f_x,\ f_y$$

$$\frac{\partial f}{\partial x} = \ln(\sin(xy)) \cdot \frac{\partial}{\partial x} \sin(xy) = \ln(\sin(xy)) \cdot \cos(xy) \cdot y$$

$$- \ln(7) \frac{\partial}{\partial x} 7$$

$$\frac{\partial f}{\partial y} = \ln(\sin(xy)) \cdot \frac{\partial}{\partial y} \sin(xy) - \ln(7) \frac{\partial}{\partial x} 7 .$$

$f(x,y,z) = x^2y + y^2z$, find Direction in which the function increases the most at $(1,2,3)$

$\vec{\nabla} f$ increases most / steepest ascent

$-\vec{\nabla} f$ decreases most / steepest descent

$\vec{\nabla} f = \langle 2xy,\ x^2 + 2yz,\ y^2 \rangle$

$\vec{\nabla} f(1,2,3) = \langle 4,\ 13,\ 4 \rangle$.