Lesson 10: Limits and Continuity (15.2)

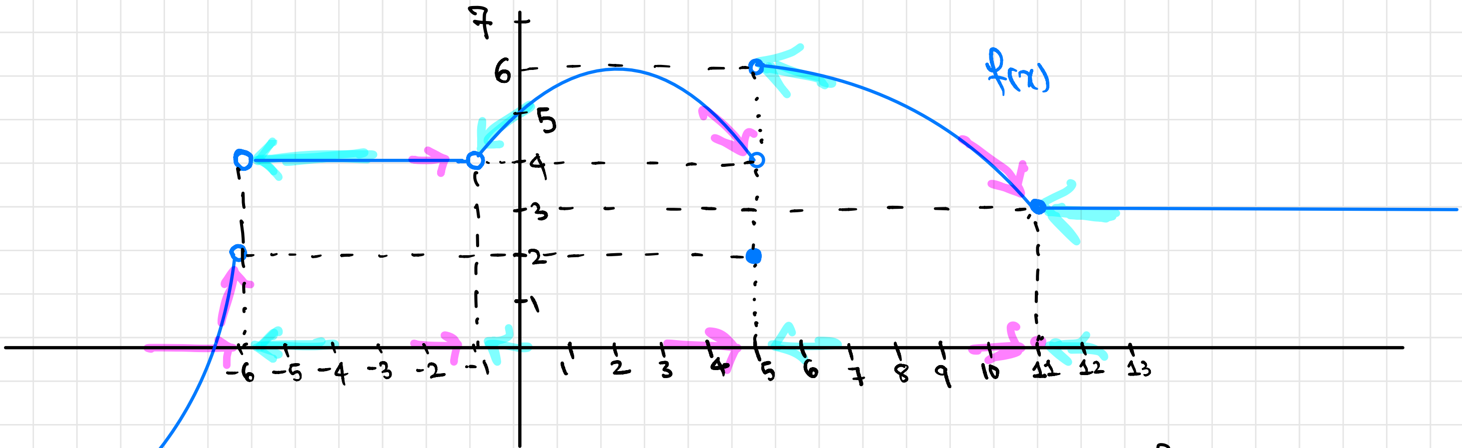
Today: Limits
Continuity

Next: Partial Derivatives

Office Hours: Monday, Friday 9:45 PM – 11:00 PM
Thursday 11:00 AM – 12:00 PM
} MATH 842
Thursday 12:15 PM – 1:15 PM (Windsor)

Warmup: Limits and Continuity (from Calculus I)

What are all the values where limit Does not exist?



two Directions to Approach -6

function Approaching Different Values

$\lim_{x \to -6} f(x)$ DNE

$\lim_{x \to -1} f(x) = 4$

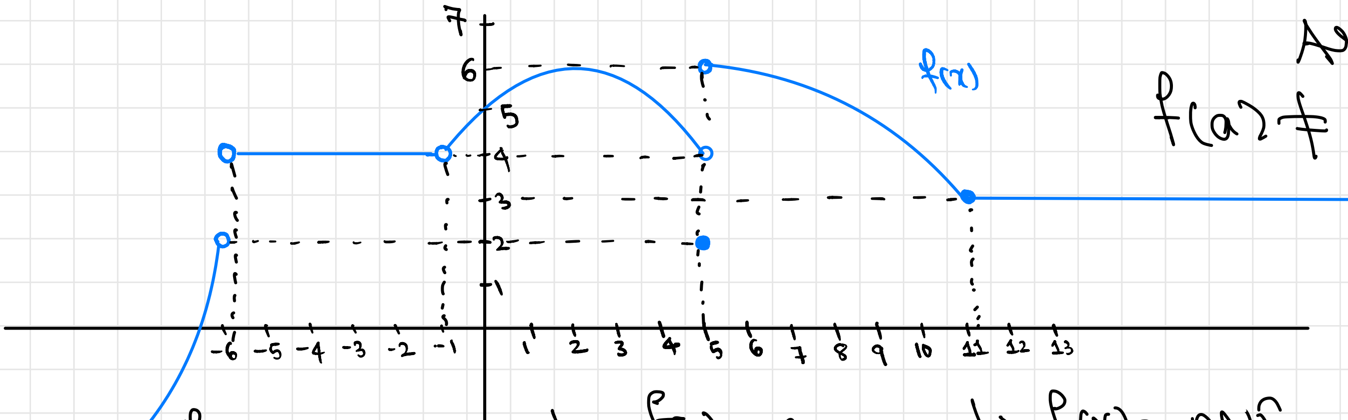
$\lim_{x \to 5} f(x)$ DNE

$\lim_{x \to 11} f(x) = 4$

What are all the values where the function is is $\underbrace{\text{not continuous}}$?

If there is hole
jump
Asymptote

$f(a) \neq \lim_{x \to a} f(x)$



$\lim_{x \to -6} f(x)$ DNE

$f(-6)$ DNE

$\lim_{x \to -1} f(x) = 4$

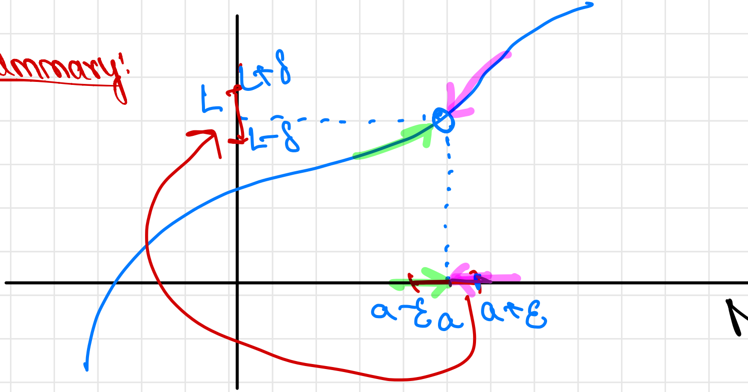
$f(-1)$ DNE

$\lim_{x \to 5} f(x) =$ DNE

$f(5) = 2$

$\lim_{x \to 11} f(x) = f(11) = 3$

Summary:



$\lim_{x\to a} f(x)$ is function value as x is close to a

exists if $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$

Note: $f(a)$ need not be defined for limit to exist

Definition of Limit: $\lim_{x\to a} f(x) = L$

if for any $\varepsilon > 0$

Suppose $a-\varepsilon < x < a+\varepsilon$

then there is $\delta > 0$ such that

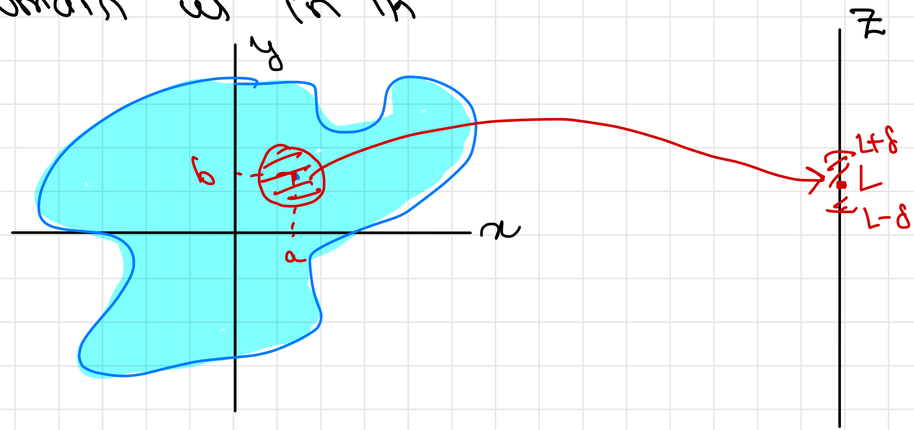
$$L-\delta < f(x) < L+\delta$$

Definition of Continuity: $f(x)$ is continuous at $x=a$

if and only if $\lim_{x\to a} f(x) = f(a)$

Limits and Continuity for two Variable functions

$f(x,y) \rightsquigarrow$ Domain is in $\mathbb{R}^2$



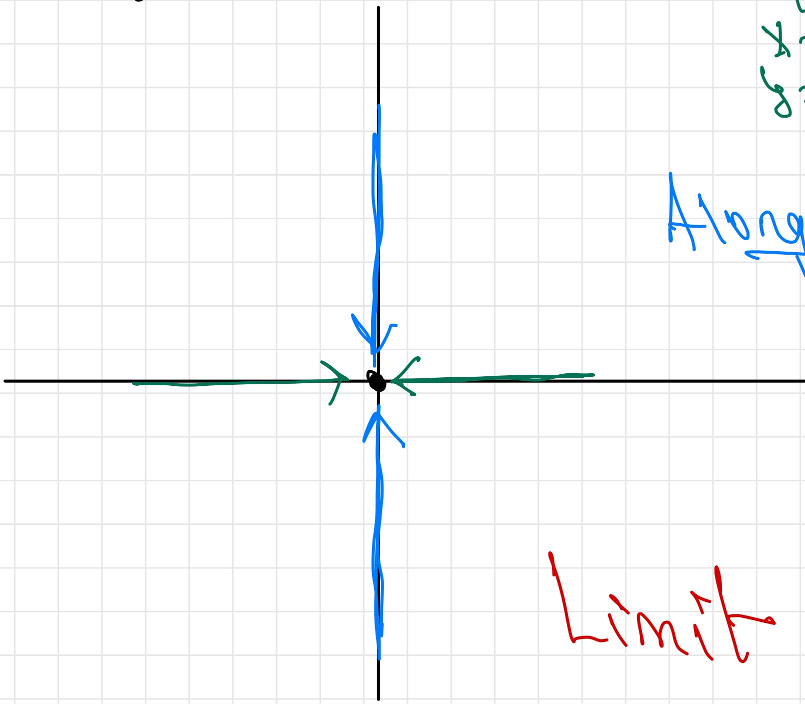
$\lim\limits_{(x,y)\to(a,b)} f(x,y) = L$ $\Big\}$ value of $f(x,y)$ is close to L when we approach (x,y) in any direction.

if for any $\varepsilon > 0$ Suppose $|(x,y) - (a,b)| < \varepsilon$

then there is $\delta > 0$ such that $|f(x,y) - L| < \delta$

$f(x,y)$ is Continuous at $(a,b]$ $\rightsquigarrow$ $\lim\limits_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

eg: $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$



Aproach along x-axis: $y=0$

$$\lim_{\substack{x\to0\\y=0}} \frac{x^2-y^2}{x^2+y^2} = \lim_{x\to0} \frac{x^2}{x^2} = 1$$

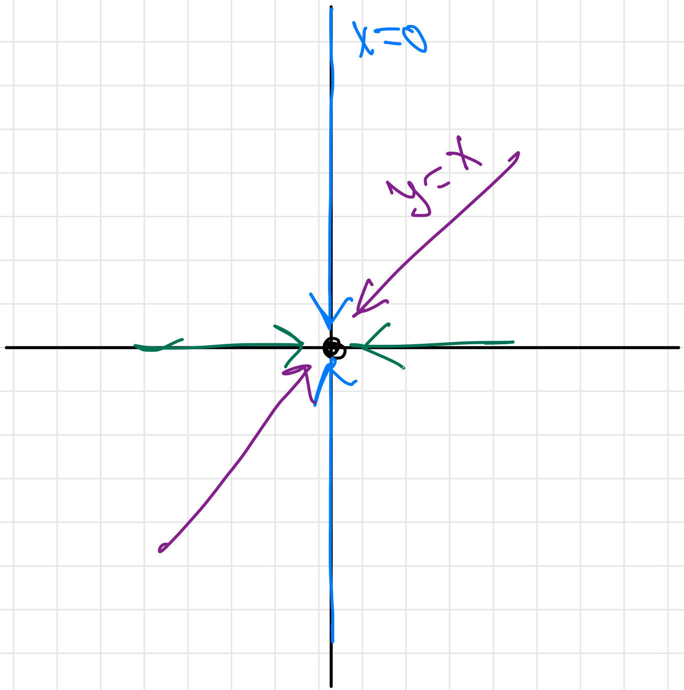
Along y-axis: $x=0$

$$\lim_{\substack{y\to0\\x=0}} \frac{x^2-y^3}{x^2+y^2} = \lim_{y\to0} \frac{-y^3}{y^2}$$

$$= -1$$

Limit DNE

eg: $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^3+y^3}$



$\underline{y=0}$:

$$\lim_{\substack{x\to 0\\ y=0}} \frac{x^2y}{x^3+y^3} = 0$$

$\underline{x=0}$:

$$\lim_{\substack{y\to 0\\ x=0}} \frac{x^2y}{x^3+y^3} = 0$$

$\underline{\underline{y=x}}$

$$\lim_{\substack{x\to 0\\ y=x}} \frac{x^2y}{x^3+y^3} = \lim_{x\to 0} \frac{x^3}{x^3+x^3} = 1/2$$

limit DNE.

Break!!

Stretch

Walk around

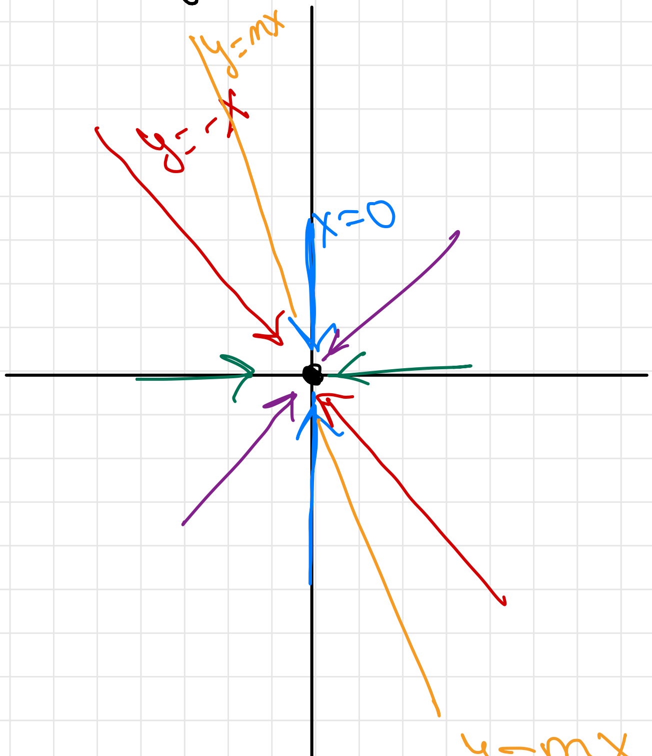
Reflect

Talk to your neighbors

have questions? Ask using iclicker

eg: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x-y}$

$\nexists$



$y=0$

$$\lim_{x\to 0} \frac{x^3}{x} = 0$$

$x=0$

$$\lim_{y\to 0} \frac{-y^3}{-y} = 0$$

$y=x$ Not in Domain

$y=-x$ $$\lim_{x\to 0} \frac{x^3-(-x)^3}{x-(-x)} = \lim_{x\to 0} \frac{2x^3}{2x} = 0$$

$y=mx,\ m\neq 1$

$$\lim_{x\to 0} \frac{x^3-m^3x^3}{x-mx} = \lim_{x\to 0} \frac{x^3(1-m^3)}{x(1-m)}$$

$$= 0$$

$\left.\dfrac{x^3-y^3}{x-y}\right\}$ simplify using a Formula $x^3-y^3=(x-y)(x^2+xy+y^2)$

or

Long Division

$$\begin{array}{r|l} & x^2+xy+y^2 \\ \hline x-y & x^3-y^3 \\ & \cancel{x^3}-x^2y \quad (-)\ (+) \\ \hline & \cancel{x^2y}-y^3 \\ & \cancel{x^2y}-xy^2 \quad (-)\ (+) \\ \hline & \cancel{xy^2}-y^3 \\ & \cancel{xy^2}-\cancel{y^3} \\ \hline & 0 \end{array}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x-y}$$

$$=\lim_{(x,y)\to(0,0)} \frac{\cancel{(x-y)}(x^2+xy+y^2)}{\cancel{(x-y)}}$$

$$=\lim_{(x,y)\to(0,0)} x^2+xy+y^2 = 0^2+0.0+0^2=0$$

plug in $(0,0)$ because x^2+xy+y^2 is continuous.

eg: $$\lim_{(x,y)\to(0,0)} \frac{\cos(7x^3y)}{8+e^{3xy^2}}$$

continuous

continuous

$\frac{\text{continuous}}{\text{continuous}}$ = continuous in the domain (denominator $\neq 0$)

$$= \frac{\cos(0)}{8+e^0}$$

$$= \frac{1}{9}.$$

Discuss with Neighbor (1 min) - Share on iClicker (1 min)

Evaluate $\lim_{x \to 0} 5\,\frac{\sin 5x}{5x}$

A) DNE

B) 1

C) 0

D) 5

E) $\frac{1}{5}$

$5x = \theta$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

$\lim_{\theta \to 0} 5\,\frac{\sin(\theta)}{\theta} = 5$

eg: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{(x^2+y^2)}$

$\displaystyle\lim_{(x,y)\to(0,0)} x^2+y^2 = 0$

$\theta = x^2+y^2$ ω $(x,y)\to 0$ then $\theta\to 0$

$\displaystyle\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

eg: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(5(x^2+y^2))}{x^2+y^2} = 5$

Exit Ticket

Should I continue to give a break?

A) Yes, but facilitate an activity

B) Yes, continue with the same

C) No, do not waste our time.

D) Fine with anything