Lesson 2 (01/14/26)

Today: Lines and Planes (13.5)

Next: Quadric Surfaces (13.6)

Announcements:

* HW Due: Lesson1, Lesson 2 (Tomorrow)
* SI Sessions
* Data Science Labs
* Lecture Notes

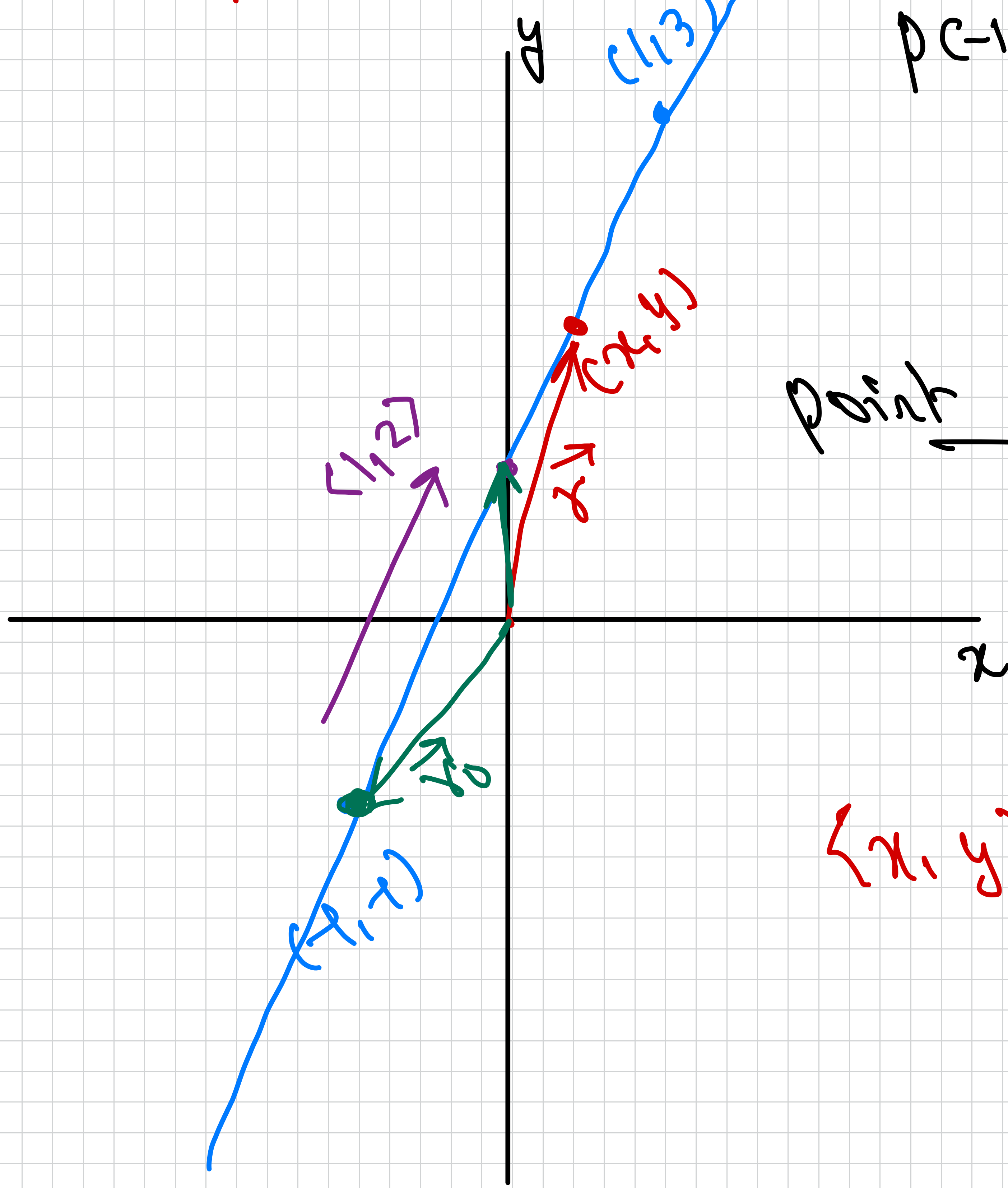
Check Bright Space Announcement

Office Hours:

Monday, Friday 9:45 AM – 11:00 AM
Thursday 11:00 AM – 12:00 PM
MATH 842

Feasting with Faculty: Thursday 12:15 PM – 1:15 PM Windsor

Warmup: Find equation of a line through $P(-1,-1)$ and $Q(1,3)$



Slope: $\dfrac{3-(-1)}{1-(-1)} = 2$

point slope form: $\dfrac{y-y_0}{x-x_0} = \text{slope}$

$$\frac{y+1}{x+1} = 2 \rightsquigarrow \boxed{y = 2x+1}$$

$$\langle x, y\rangle = \langle x, 2x+1\rangle = t\langle 1,2\rangle + \langle 0,1\rangle$$

$\langle 1,2\rangle$: parallel to line "Direction"

$\langle 0,1\rangle$: on line

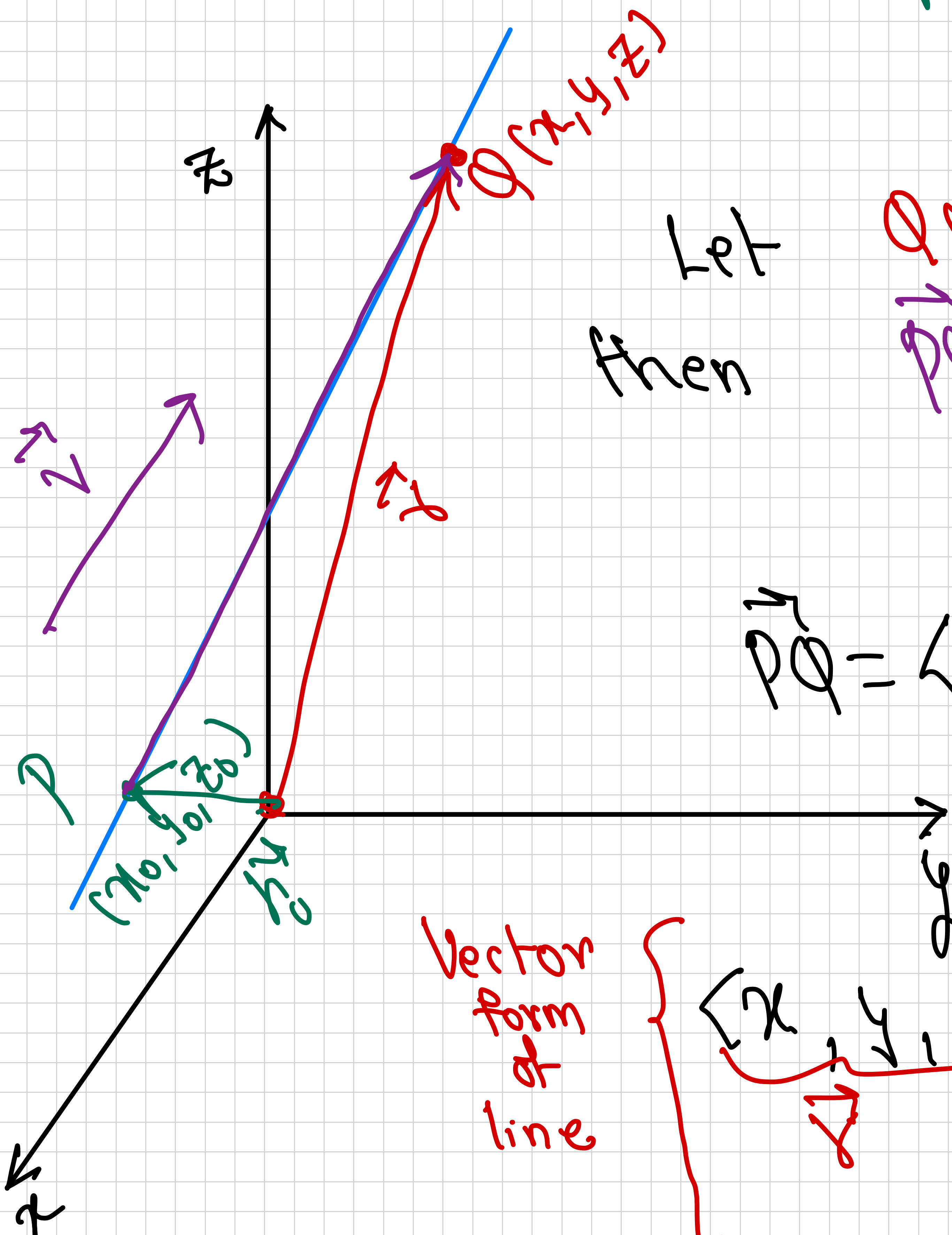
Lines in 3D: Need a point on line & "Slope"

$P(x_0, y_0, z_0)$

↓

Direction vector

$\vec{v} = \langle a, b, c \rangle$



Let $Q(x, y, z)$ be a point on line

then $\vec{PQ}$ is parallel to $\vec{v}$

$$\vec{PQ} = t\,\vec{v}$$

$$\vec{PQ} = \langle x - x_0, y - y_0, z - z_0 \rangle = t\langle a, b, c \rangle$$

re arrange terms

Vector form of line

$$\underbrace{\langle x, y, z \rangle}_{\vec{r}} = t\underbrace{\langle a, b, c \rangle}_{\vec{v}} + \underbrace{\langle x_0, y_0, z_0 \rangle}_{\vec{r}_0}$$

Parametric and Symmetric form of Line

$$\vec{r} = t\vec{v} + \vec{r}_0 \quad \Big\}\ \text{vector form}$$

$$\langle x, y, z\rangle = t\langle a, b, c\rangle + \langle x_0, y_0, z_0\rangle$$

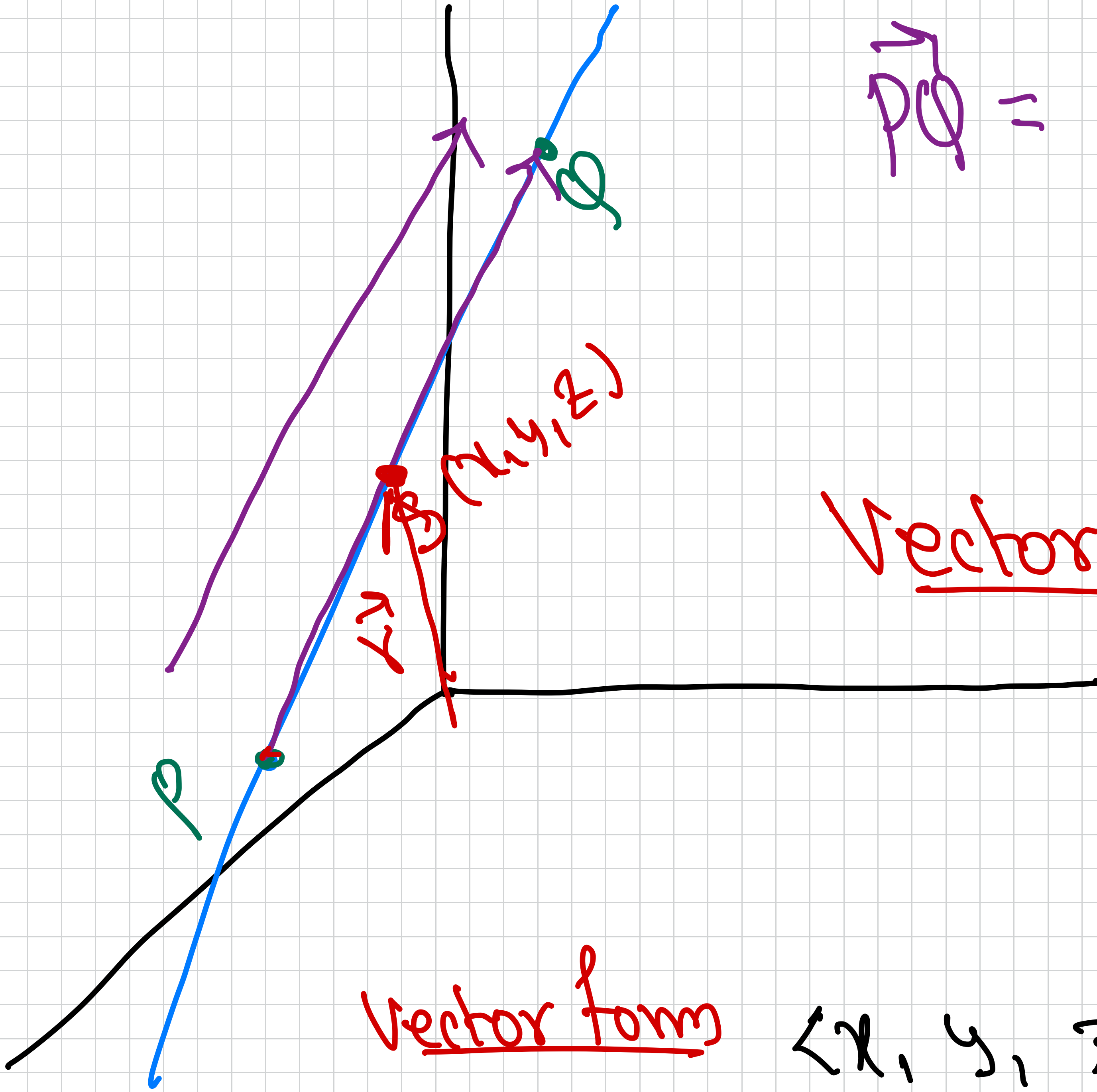
$$\langle x, y, z\rangle = \langle ta + x_0,\ tb + y_0,\ tc + z_0\rangle$$

$$\left.\begin{aligned} x &= ta + x_0 \\ y &= tb + y_0 \\ z &= tc + z_0 \end{aligned}\right\}\ \text{Parametric form.}$$

Solve for t

$$t = \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \quad \Big\}\ \text{Symmetric form or}$$

eg: Find equation of line passing through $\underbrace{(1,0,2)}_{P}$ & $\underbrace{(-2,4,-3)}_{Q}$.



$$\vec{PQ} = \langle -2-1,\ 4-0,\ -3-2 \rangle = \langle -3,\ 4,\ -5 \rangle$$

$$\vec{v} = \langle -3, 4, -5 \rangle \Big\}\ \text{direction vector}$$

Vector form: $\vec{r} = \underbrace{t\,\vec{v}}_{\text{Direction vector}} + \vec{r}_0$

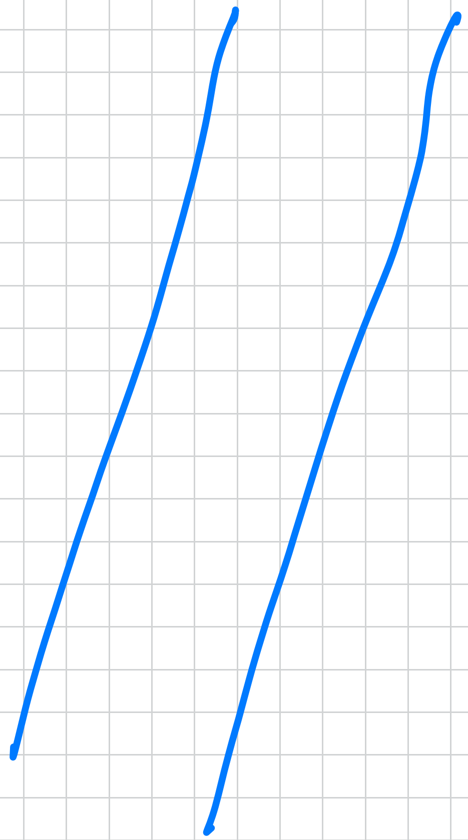
one point "can choose any of the two"

Vector form $\langle x, y, z \rangle = t\,\langle -3, 4, -5 \rangle + \langle -2, 4, -3 \rangle$

Parametric: $x = -3t-2,\ y = 4t+4,\ z = -5t-3$

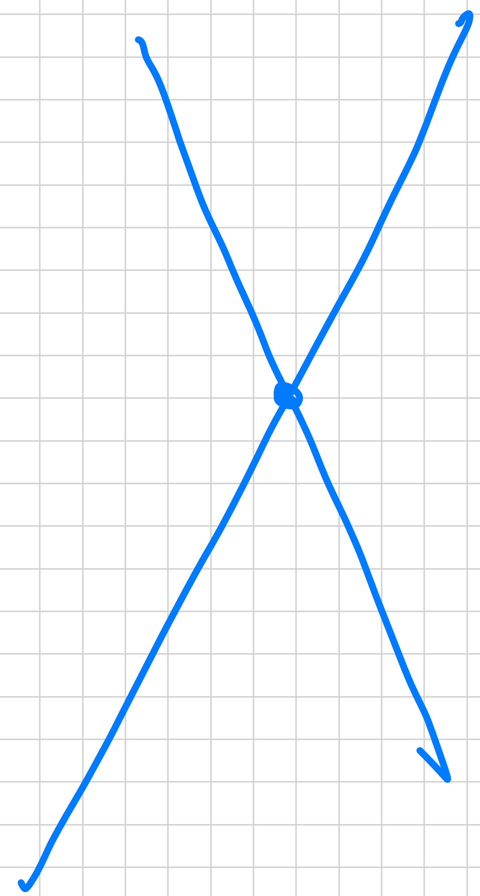
Symmetric: $$\frac{x+2}{-3} = \frac{y-4}{4} = \frac{z+3}{-5}$$

Parallel



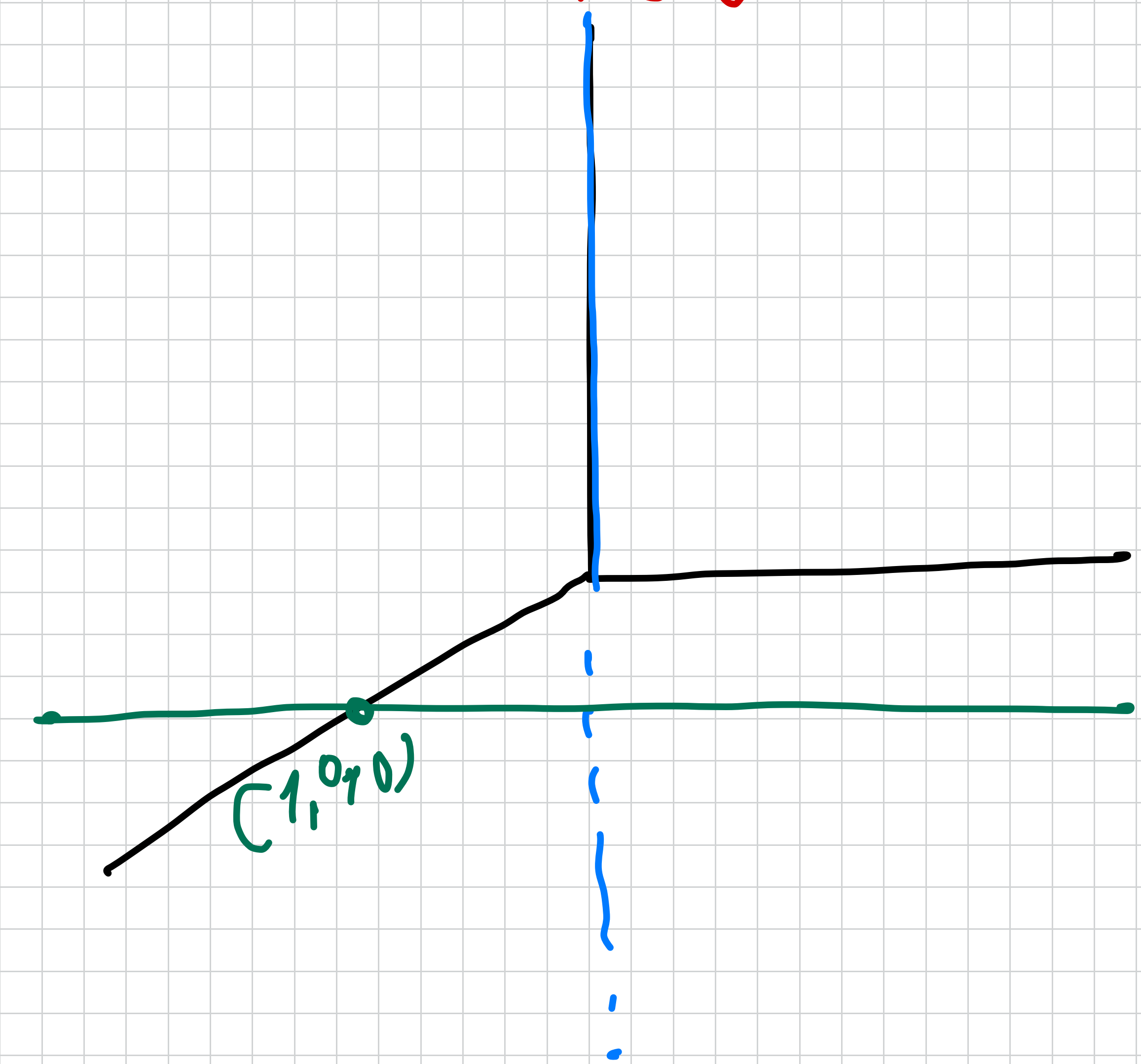
Direction vector are parallel

Intersecting



have a point in common

Skew



$l_1: \ t\langle 0,0,1\rangle$

$l_2: \ t\langle 0,1,0\rangle + \langle 1,0,0\rangle$

↳ No Common Point

↳ Direction vectors not parallel

Break

Stretch

Reflect

Ask questions

talk to your neighbor

. . . .

Next: Planes in 3D.

Turn to your neighbor & Discuss these Questions:

① How do we find a vector orthogonal to $\vec{u}$ & $\vec{v}$?

$$\vec{u} \times \vec{v} \perp \vec{u}$$

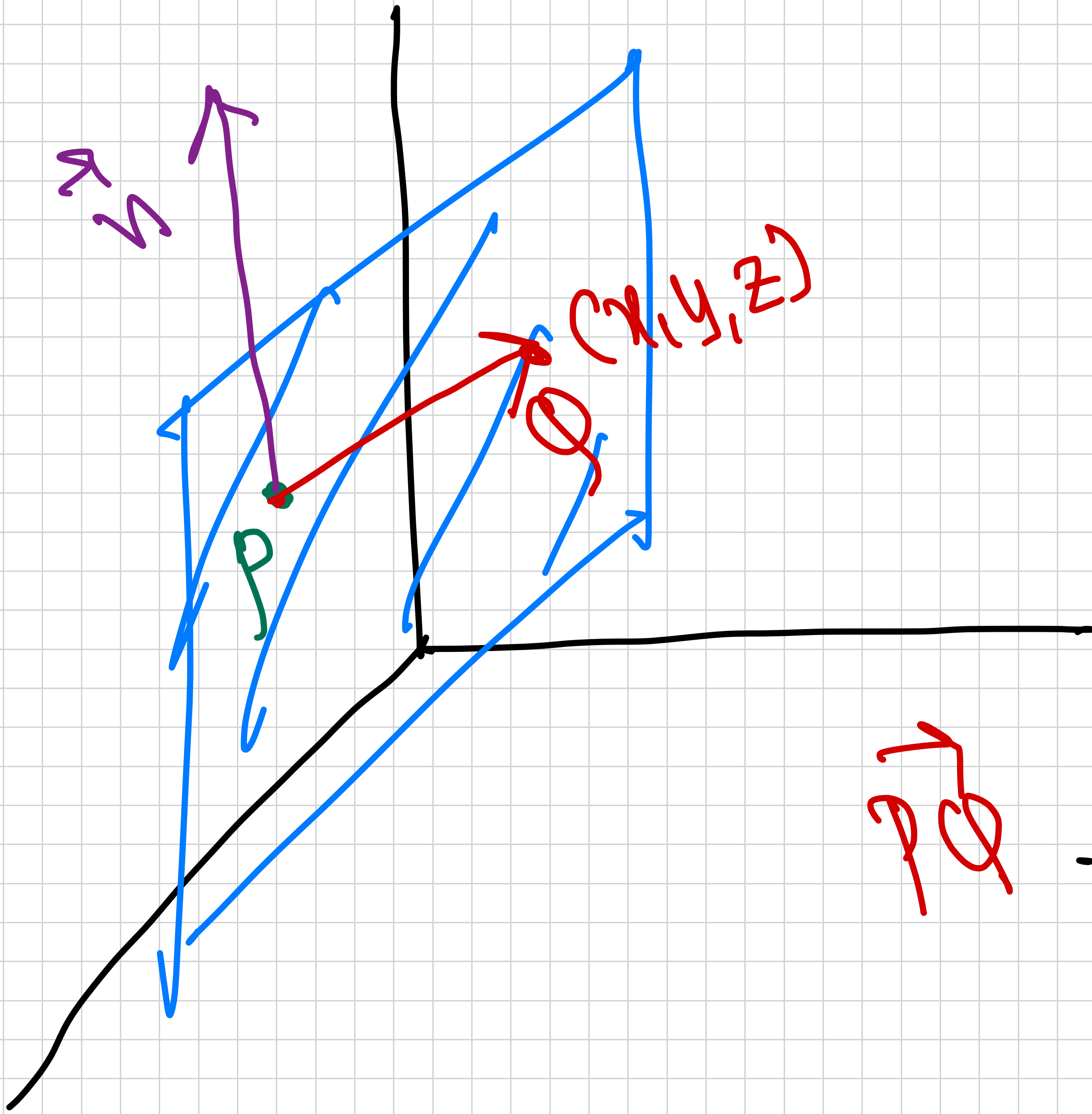
$$\vec{u} \times \vec{v} \perp \vec{v}$$

② How do we check if $\vec{u}$ and $\vec{v}$ are orthogonal?

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta = 0 \rightsquigarrow \theta = \frac{\pi}{2}$$

$$\vec{u} \perp \vec{v}$$

Plane: Need a point on plane $P(x_0, y_0, z_0)$ & Normal Vector $\vec{n} = \langle a, b, c \rangle$ is perpendicular to all vectors on plane



$Q(x, y, z)$ } Any point in plane

$\vec{PQ} \perp \vec{n} \rightsquigarrow (\vec{PQ}) \cdot \vec{n} = 0$

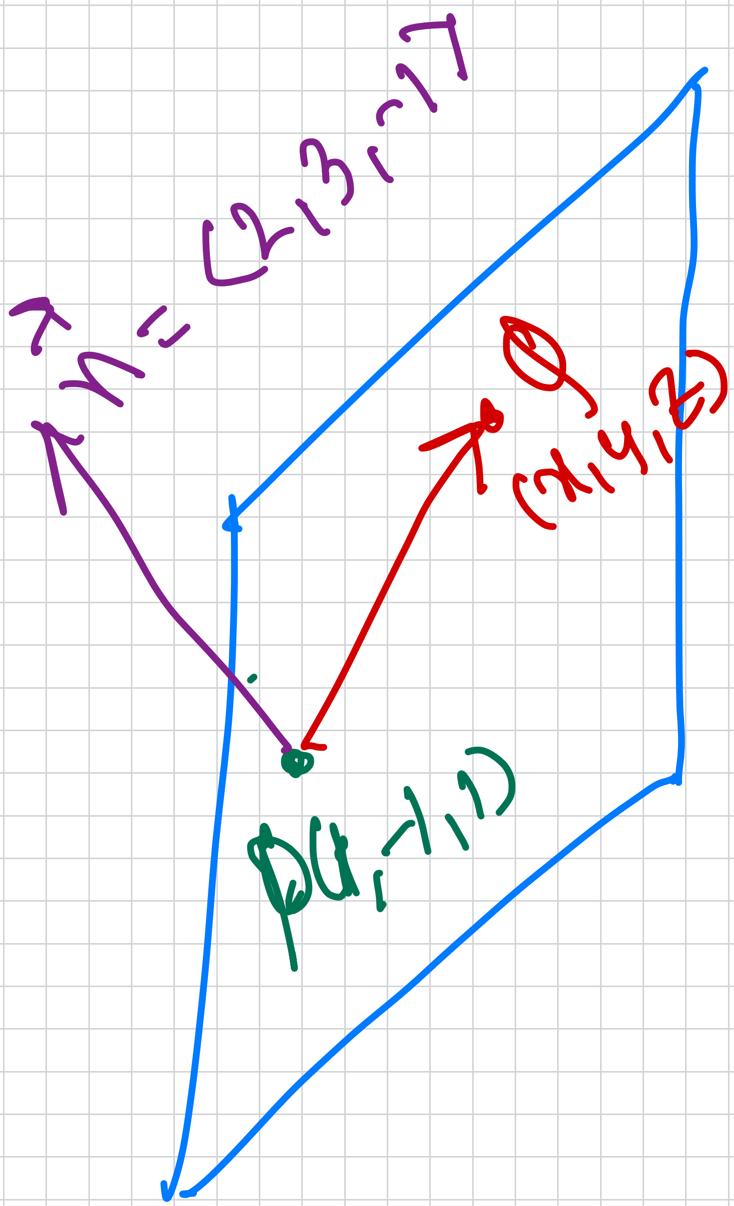
$\langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle a, b, c \rangle = 0$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax + by + cz = ax_0 + by_0 + cz_0$$

} Equation of Plane

Find equation of plane passing through $(1,-1,1)$ with Normal vector $\langle 2,3,-1\rangle$



$$\vec{PQ} \perp \vec{n}$$

$$\langle x-1, y+1, z-1\rangle \perp \langle 2,3,-1\rangle$$

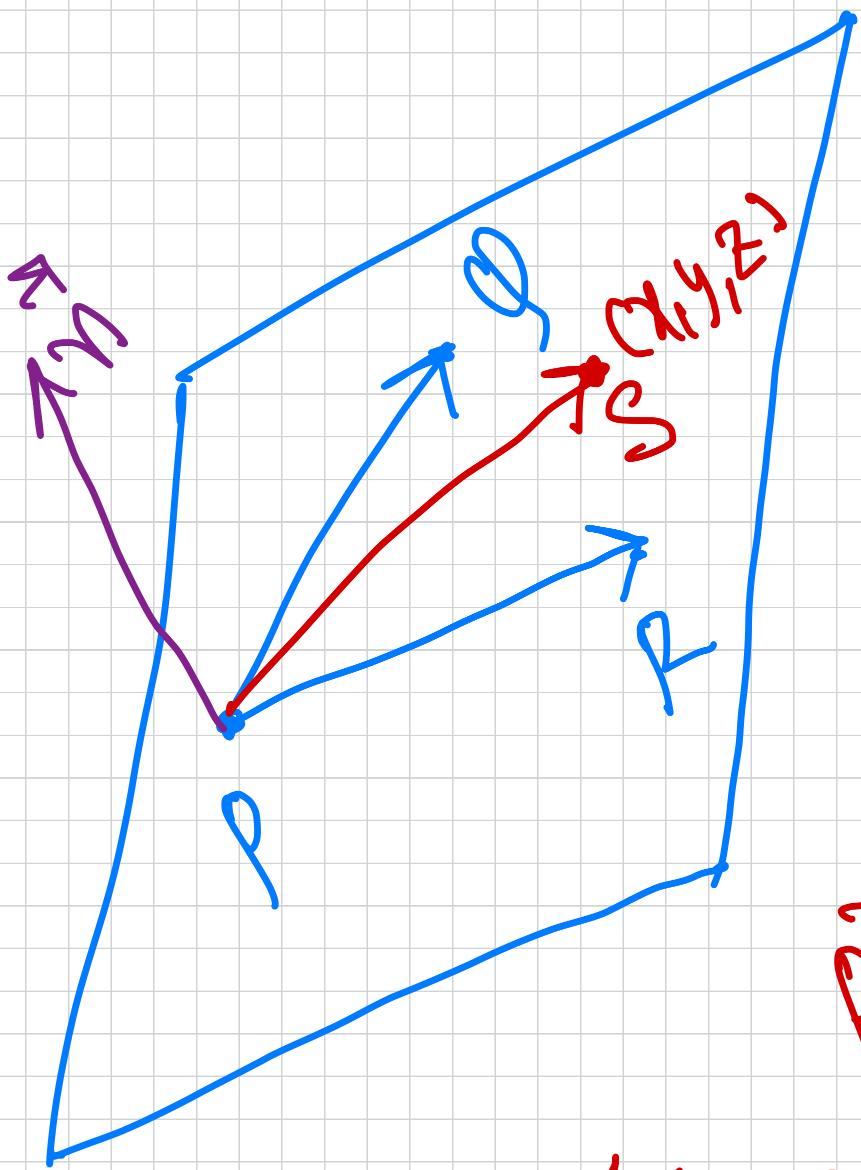
$$\langle x-1, y+1, z-1\rangle \cdot \langle 2,3,-1\rangle = 0$$

$$2(x-1)+3(y+1)-(z-1)=0$$

$$2x-2+3y+3-z+1=0$$

$$\boxed{2x+3y-z=-2}$$

Find equation of Plane containing $P(2,3,-1)$, $Q(3,5,-1)$ and $R(6,2,0)$



$\vec{PQ} \times \vec{PR}$ is $\perp$ to plane

$$\vec{n} = \vec{PQ} \times \vec{PR}$$

$$\vec{PQ} = \langle 1, 2, 0 \rangle$$

$$\vec{PR} = \langle 4, -1, 1 \rangle$$

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 4 & -1 & 1 \end{vmatrix} = \langle 2, -1, -9 \rangle$$

$$\vec{PS} \perp \vec{n}$$

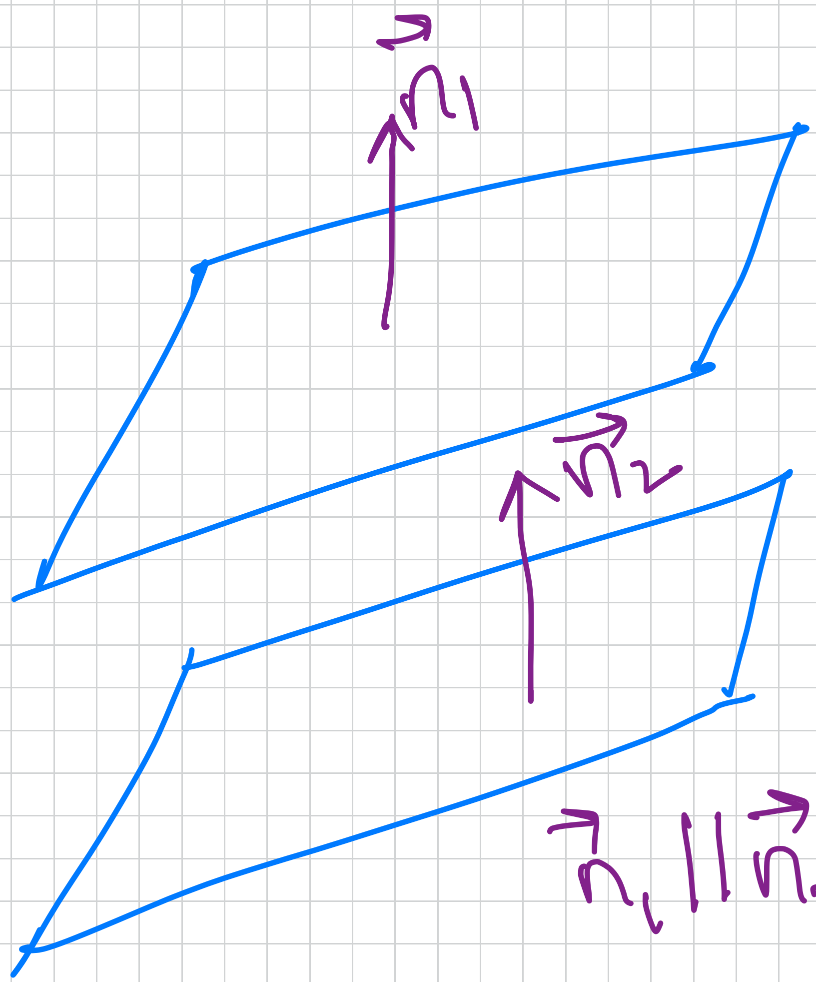
$$\langle x-2, y-3, z+1 \rangle \cdot \langle 2, -1, -9 \rangle = 0$$

$$2(x-2) - (y-3) - 9(z+1) = 0$$

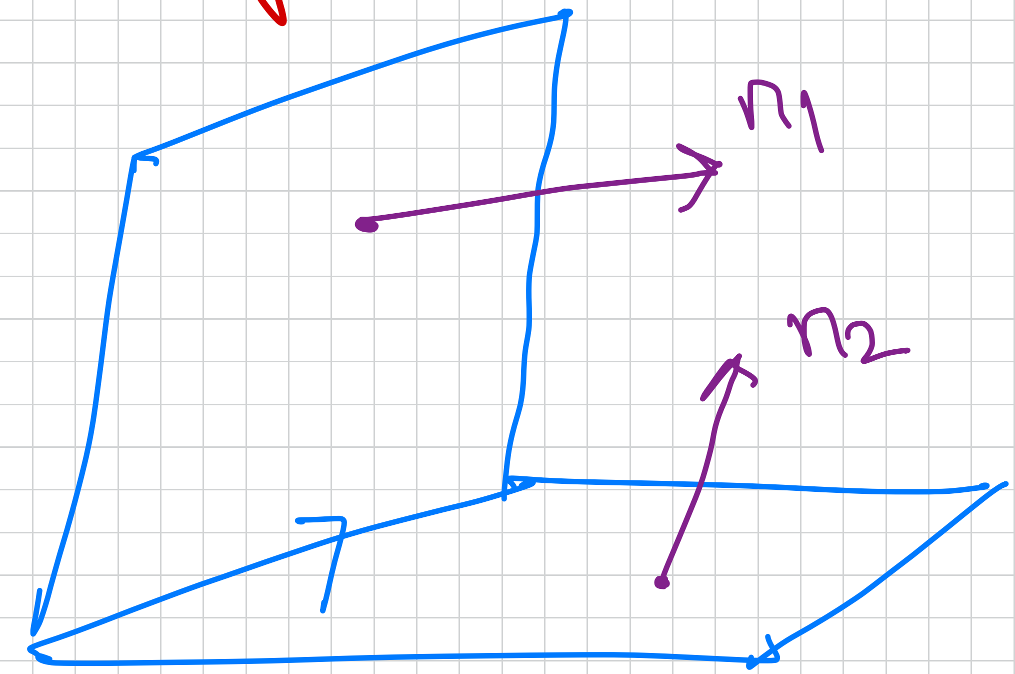
$$2x - y - 9z = 10$$

instead of P you can choose Q or S. you will get same answer.

Parallel and Orthogonal Planes



$\vec{n}_1 \parallel \vec{n}_2$



$n_1 \perp n_2$.

planes are parallel if Normal Vectors are parallel

planes are orthogonal if Normal vectors are orthogonal.

Examples Not worked out in class

eg① $\vec{r}_1(t) = \langle 2t+3, 4t+2, 3t+5 \rangle$
$\vec{r}_2(t) = \langle s+2, 3s-1, -5s+10 \rangle$
find point of intersection

$2t+3 = s+2$
$\Rightarrow \boxed{s = 2t+1}$

$4t+2 = 3s-1$
$= 3(2t+1)-1$
$4t+\cancel{2} = 6t+\cancel{2}$

$$\boxed{\begin{array}{l} t=0 \\ s = 2t+1 \xrightarrow[t=0]{} s=1 \end{array}}$$

$t=0 \rightsquigarrow 3t+5=5$
$s=1 \rightsquigarrow -5s+10=5$

point of intersection $= \langle 3, 2, 5 \rangle$ } get by plugging in $t=0$ in $\vec{r}_1$ or $s=1$ in $\vec{r}_2$

eg ②: $x+3y-2z=1$, $x+y+z=0$ } Find equation of the line of intersection.

Solve two variables interms of the third variable

for example write x & y interms of z

$$\begin{array}{r} x+3y-2z=1 \\ x+y+z=0 \\ (-)\ (-)\ (-) \\ \hline 2y-3z=1 \end{array} \rightsquigarrow y=\frac{1}{2}+\frac{3}{2}z$$

plug into one of the equations

$$x+y+z=0$$

$$x+\frac{1}{2}+\frac{3}{2}z+z=0 \rightsquigarrow x=\frac{-1}{2}-\frac{5}{2}z$$

$$\langle x,y,z\rangle=\left\langle \frac{-1}{2}-\frac{5}{2}z,\ \frac{1}{2}+\frac{3}{2}z,\ z\right\rangle=\underbrace{z\left\langle \frac{-5}{2},\frac{3}{2},1\right\rangle+\left\langle \frac{-1}{2},\frac{1}{2},0\right\rangle}_{\text{equation of the line of intersection}}$$