Lesson 20: Double integrals in Polar Coordinates

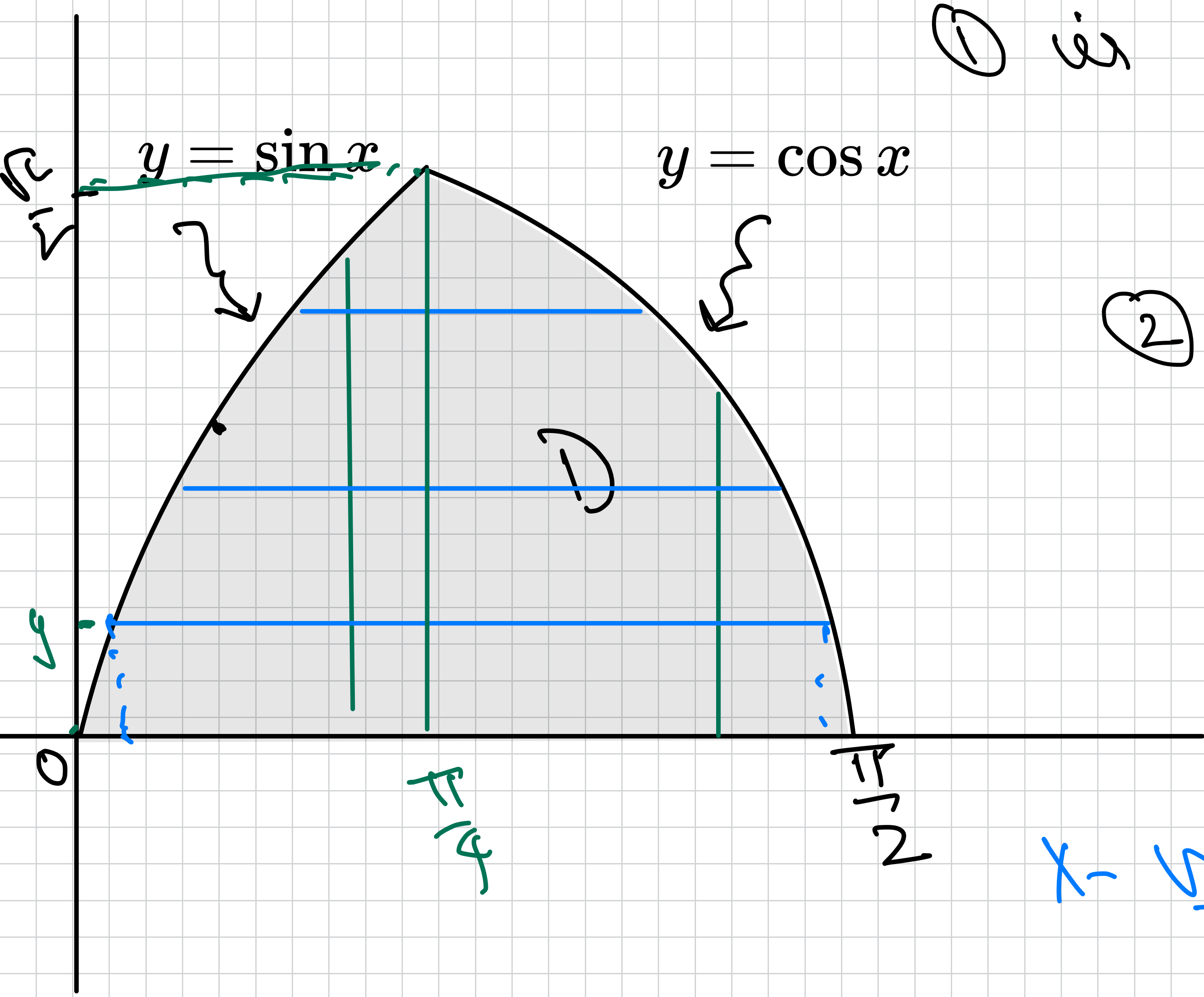
Announcements:

* Exam 1 scores posted on Brightspace
* Exam 1 key will be posted soon
* Feasting with faculty to continue from tomorrow (12:15pm - 1:15pm Windsor)

Office Hours: Monday, Friday: 9:45am - 11:15 am

Thursday: 11am - 12pm

Review: Double integrals



① is D a type I or type II Region?

② $\iint_D f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy$

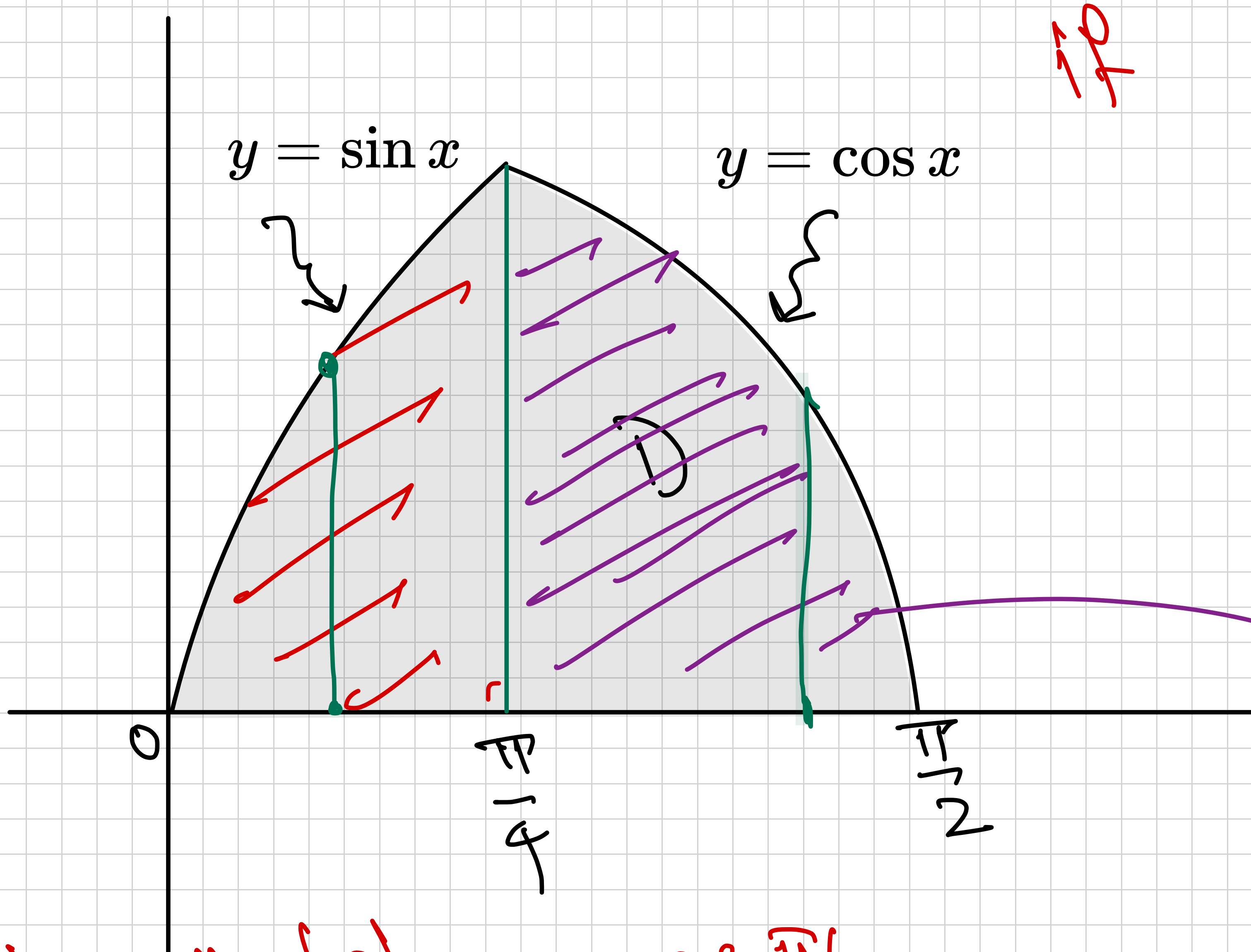
find a, b, c, d.

fix y between 0 & $\frac{\sqrt{2}}{2}$

x- value: Left curve $x = \sin^{-1} y$

Right curve $x = \cos^{-1} y$

$$\int_0^{\frac{\sqrt{2}}{2}} \int_{\sin^{-1} y}^{\cos^{-1} y} f(x,y)\,dx\,dy$$



if you want to integrate w.r.t y first?

fix x b/w $\pi/4$ & $\pi/2$

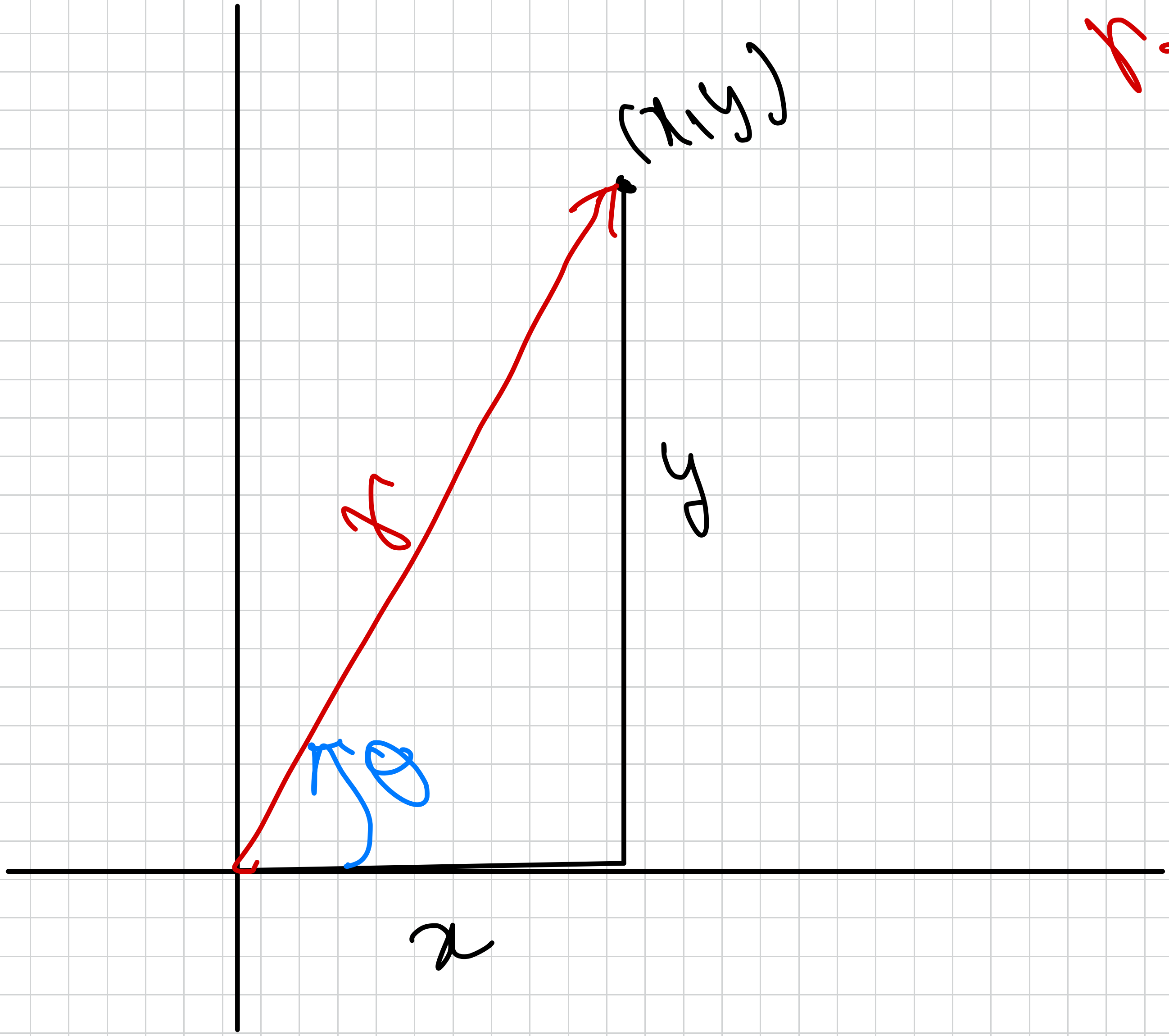
$0 \le y \le \cos x$

fix x between 0 & $\pi/4$

$0 \le y \le \sin x$

$$\int_0^{\pi/4} \int_0^{\sin x} f(x,y)\, dy\, dx + \int_{\pi/4}^{\pi/2} \int_0^{\cos x} f(x,y)\, dy\, dx$$

Review: Polar Coordinates



r = distance b/w $(0,0)$ & (x,y)

$$r = \sqrt{x^2+y^2}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

θ = Angle b/w the x-axis & Ray connecting $(0,0)$, (x,y)

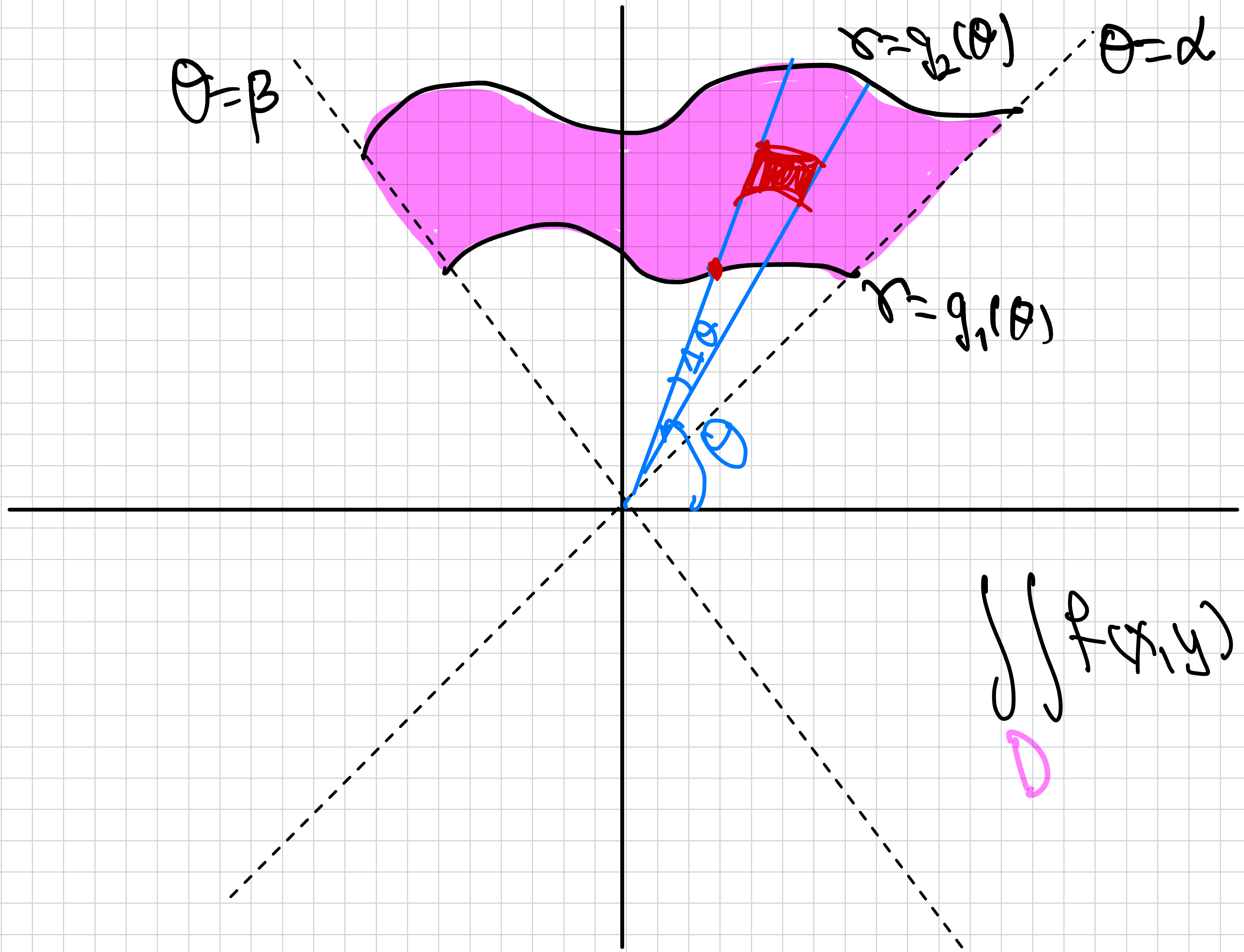
Clock wise is -ve

Counter clock wise is +ve

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Double integrals in polar coordinates

$$\iint_D f(x,y)\,dA$$



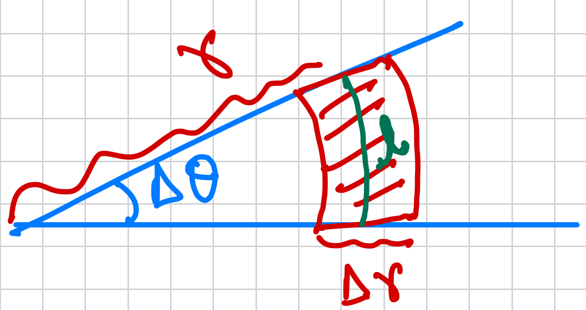
fix θ b/w α & β

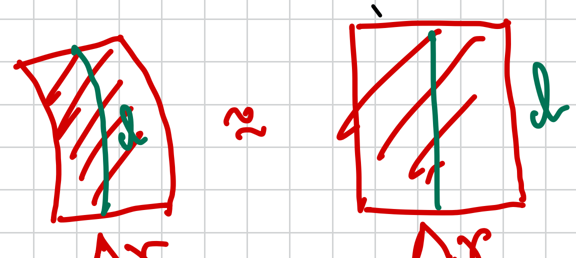
$$g_1(\theta) \le r \le g_2(\theta)$$

$$\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} f(r\cos\theta, r\sin\theta)\,\boxed{dA}$$

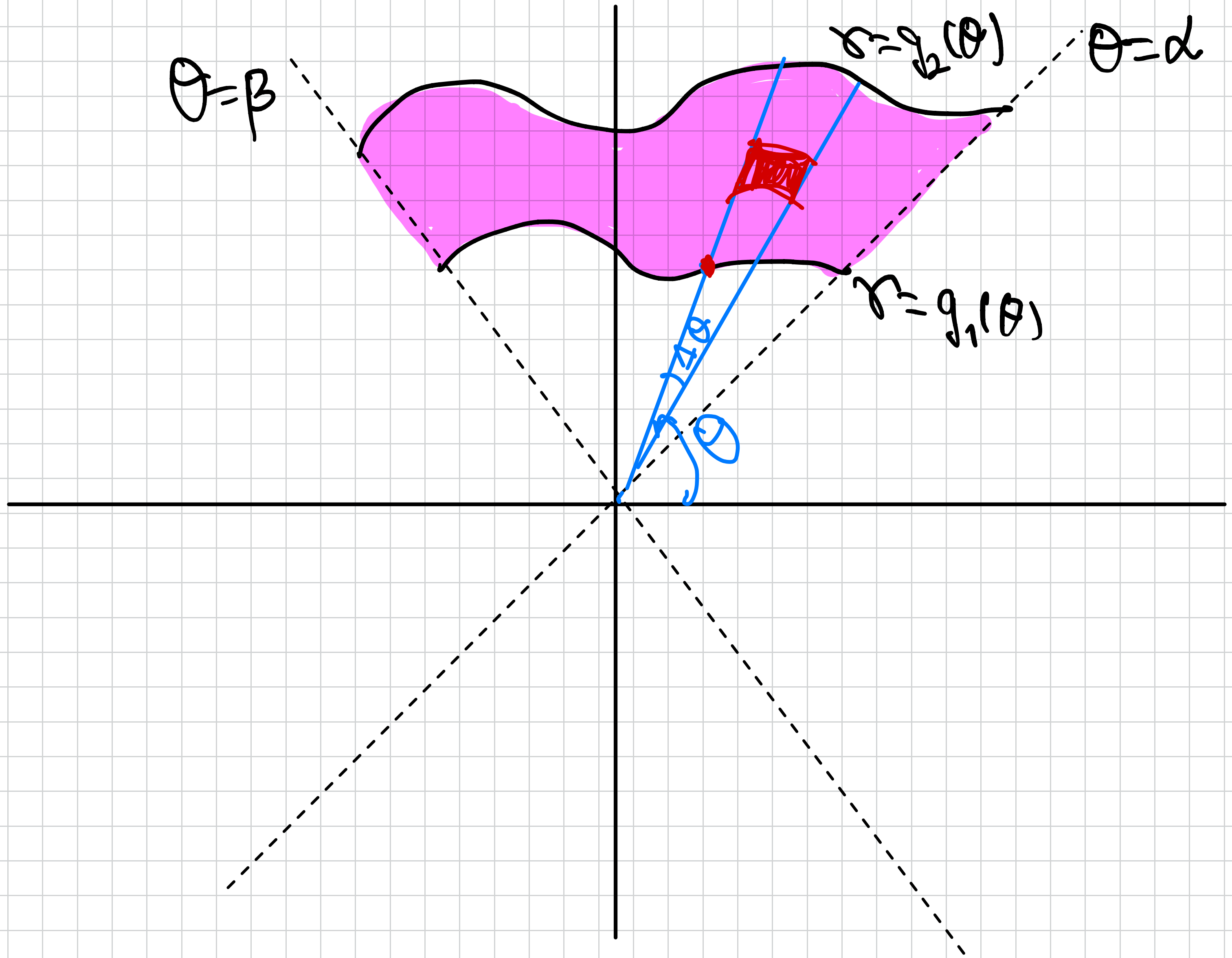
?

infinitesimal Area



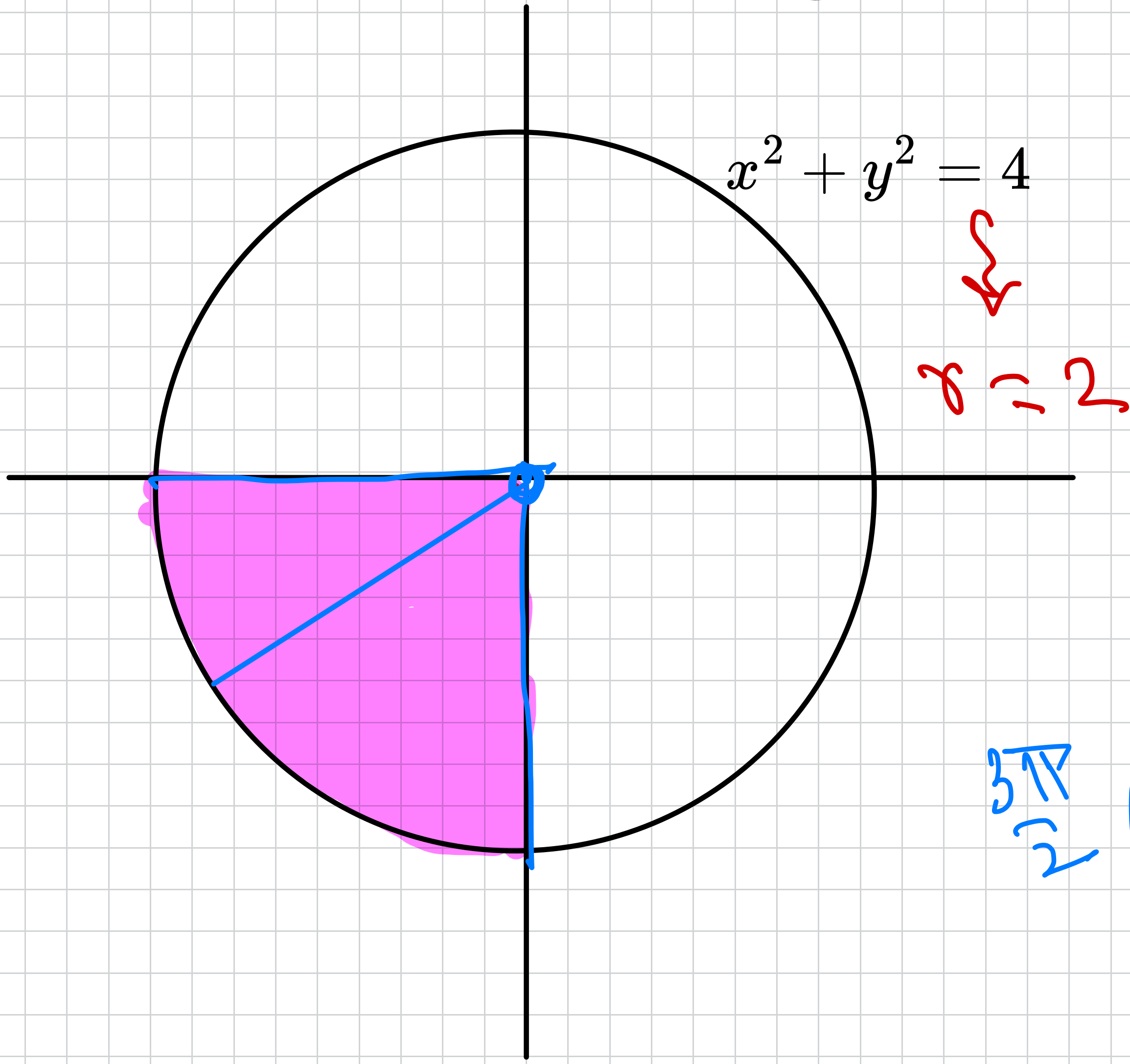


$$\left.\begin{aligned} \Delta A &= l\,\Delta r \\ l &= r\cdot\Delta\theta \end{aligned}\right\} \quad \begin{aligned} \Delta A &= r\,\Delta r\,\Delta\theta \\ dA &= \underline{\underline{r\,dr\,d\theta}} \end{aligned}$$



$$\iint_D f(x,y)\,dA = \int_{\alpha}^{\beta}\int_{g_1(\theta)}^{g_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

eg: Evaluate $\iint_D \sin(x^2+y^2)\,dA$, D is the shaded Region below



fix θ in the Region then

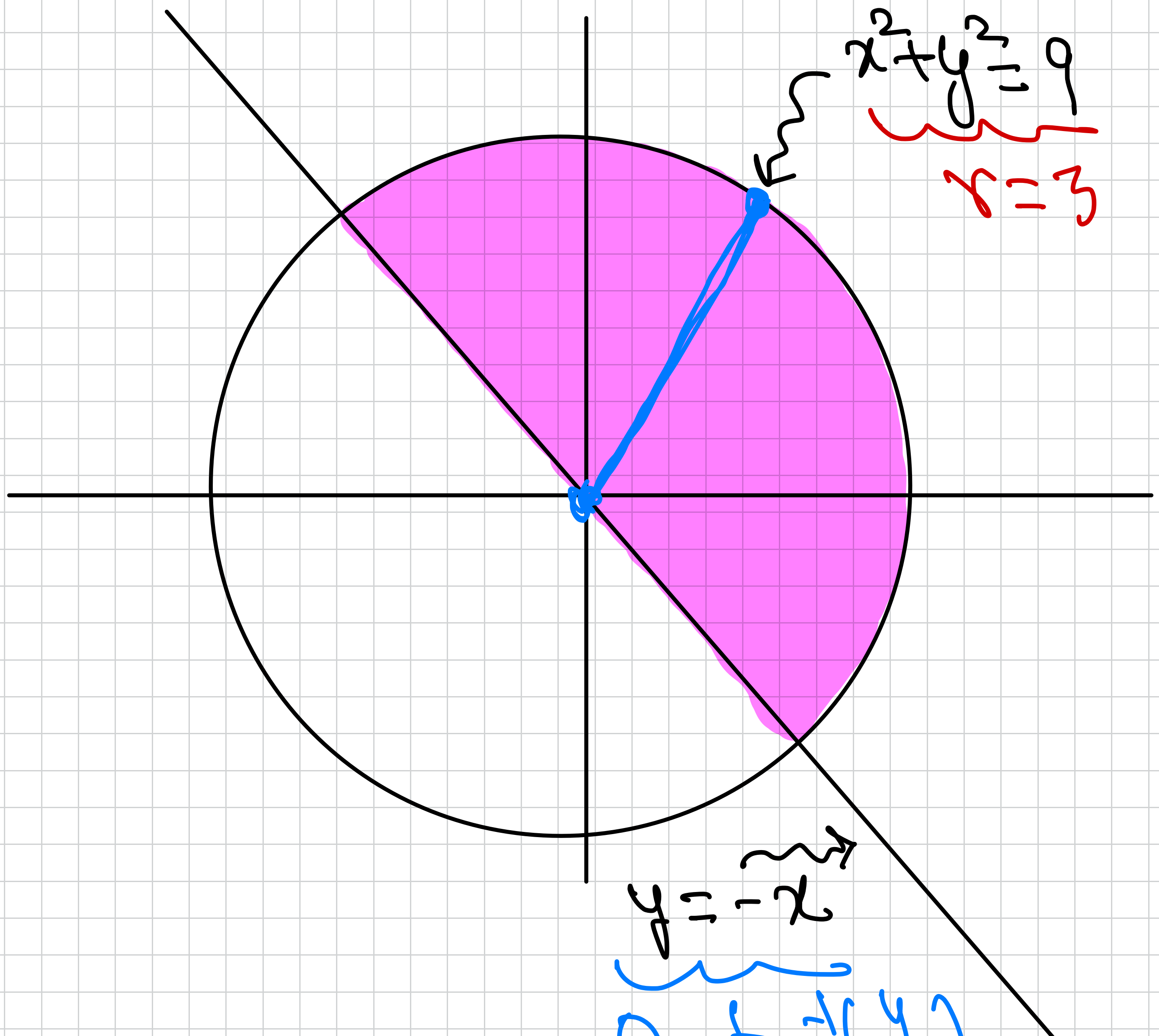
$0 \le r \le 2$

$\pi \le \theta \le \frac{3\pi}{2}$ or $-\pi \le \theta \le -\pi/2$

$$\int_{\pi}^{\frac{3\pi}{2}} \int_0^2 \sin(r^3)\, r\,dr\; d\theta$$

$$\int_{\pi}^{3\pi/2} -\frac{\cos(r^2)}{2}\Big|_0^2 \, d\theta = \left(\frac{-\cos 4 + 1}{2}\right)\frac{\pi}{2}.$$

eg: evaluate $\iint_D e^{x^2+y^2}\, dA$, D is the shaded Region below.



$\underbrace{y=-x}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}(-1)$

2nd Quadrant is $\frac{3\pi}{4}$

4th Quadrant is $-\frac{\pi}{4}$ or $\frac{7\pi}{4}$

$\underbrace{\text{fix } \theta}$, $0 \le r \le 3$

$0 \le \theta \le \frac{3\pi}{4}$ and $\frac{7\pi}{4} \le \theta \le 2\pi$

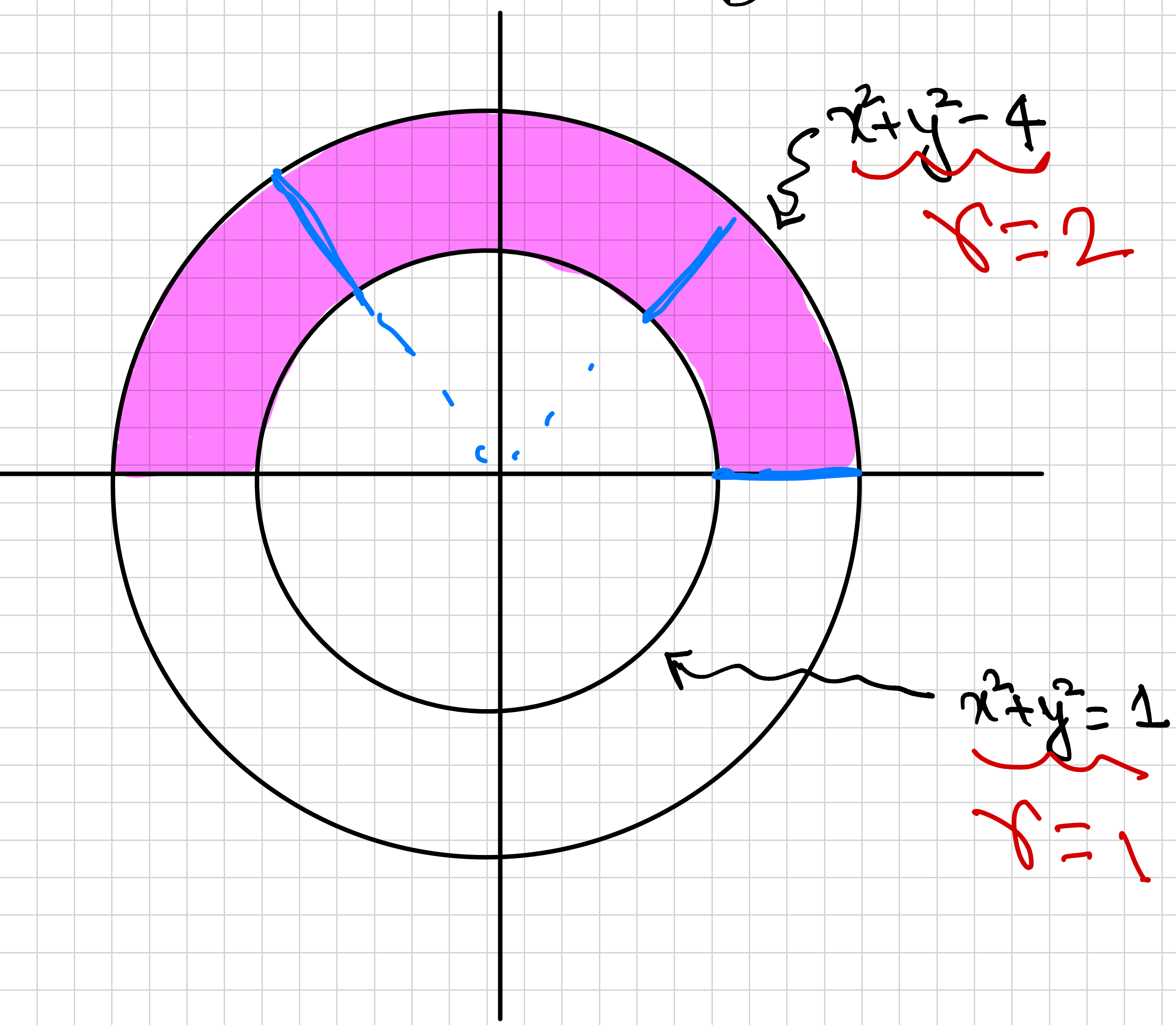
same as

$-\pi/4 \le \theta \le 0$

is $-\pi/4 \le \theta \le 3\pi/4$

$$\boxed{\int_{-\pi/4}^{3\pi/4} \int_0^3 e^{r^2} \cdot r \, dr \, d\theta}$$

eg: Evaluate $\iint_D 4x+3y \, dA$, D is the Shaded Region below.



$\underbrace{\text{fix } \theta}_{\text{b/w } 0 \text{ \& } \pi}$, $1 \le r \le 2$

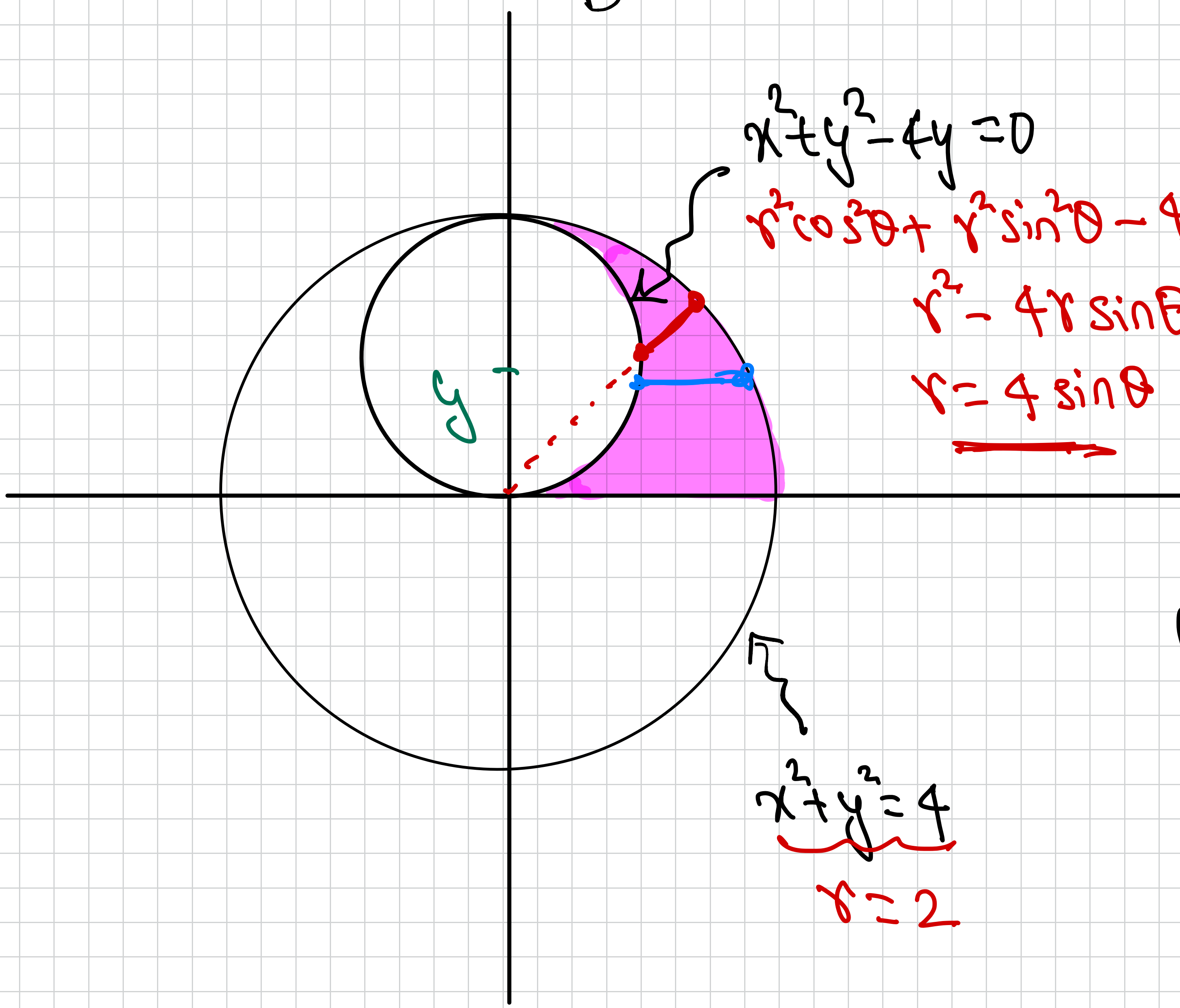
$$\int_0^{\pi} \int_1^2 (4r\cos\theta + 3r\sin\theta) r \, dr \, d\theta$$

$$\int_0^{\pi} \int_1^2 r^2 [4\cos\theta + 3\sin\theta] \, dr \, d\theta$$

$$\int_0^{\pi} \frac{7}{3} [4\cos\theta + 3\sin\theta] \, d\theta$$

$$= \frac{7}{3} [4\sin\theta - 3\cos\theta] \Big|_0^{\pi} = \underline{\underline{14.}}$$

eg: Evaluate $\iint_D y\,dA$, D is the shaded Region below.



fix θ

$\rightsquigarrow$ $4\sin\theta \le r \le 2$

$0 \le \theta \le \pi/2$

$$\iint_D y\,dA = \int_0^{\frac{\pi}{2}} \int_{4\sin\theta}^{2} (r\sin\theta)\, r\,dr\,d\theta$$