

Lesson 3: Cylinders and Quadric Surfaces (13.6) - Part I

Warmup Questions (think - pair - share)

① What does $x=2$ represent?

plane

line

point

② What does $x^2 + y^2 = 1$ represent?

circle

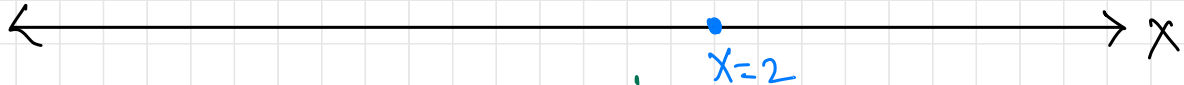
cylinder

- No context
in
my question

- unless specified
Assume $\mathbb{R}^3$

$$X = 2$$

in $\mathbb{R}^1$:



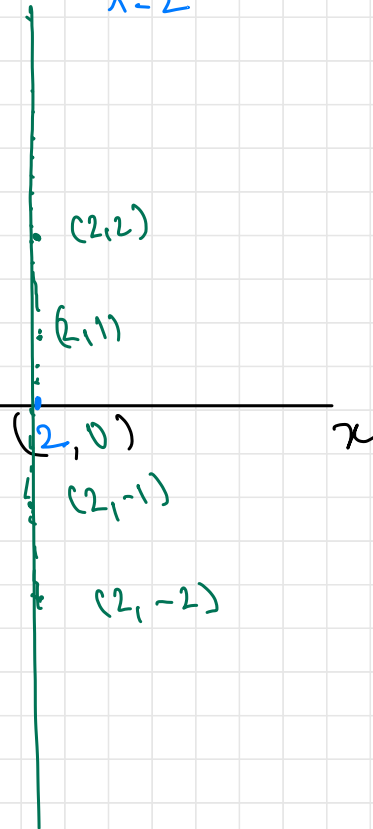
in $\mathbb{R}^2$:
xy plane

$$X = 2$$

y - can be Anything
"Free"

Stack all the points
in
y-direction

⇒ we get a line.



$X=2$ in $\mathbb{R}^3$

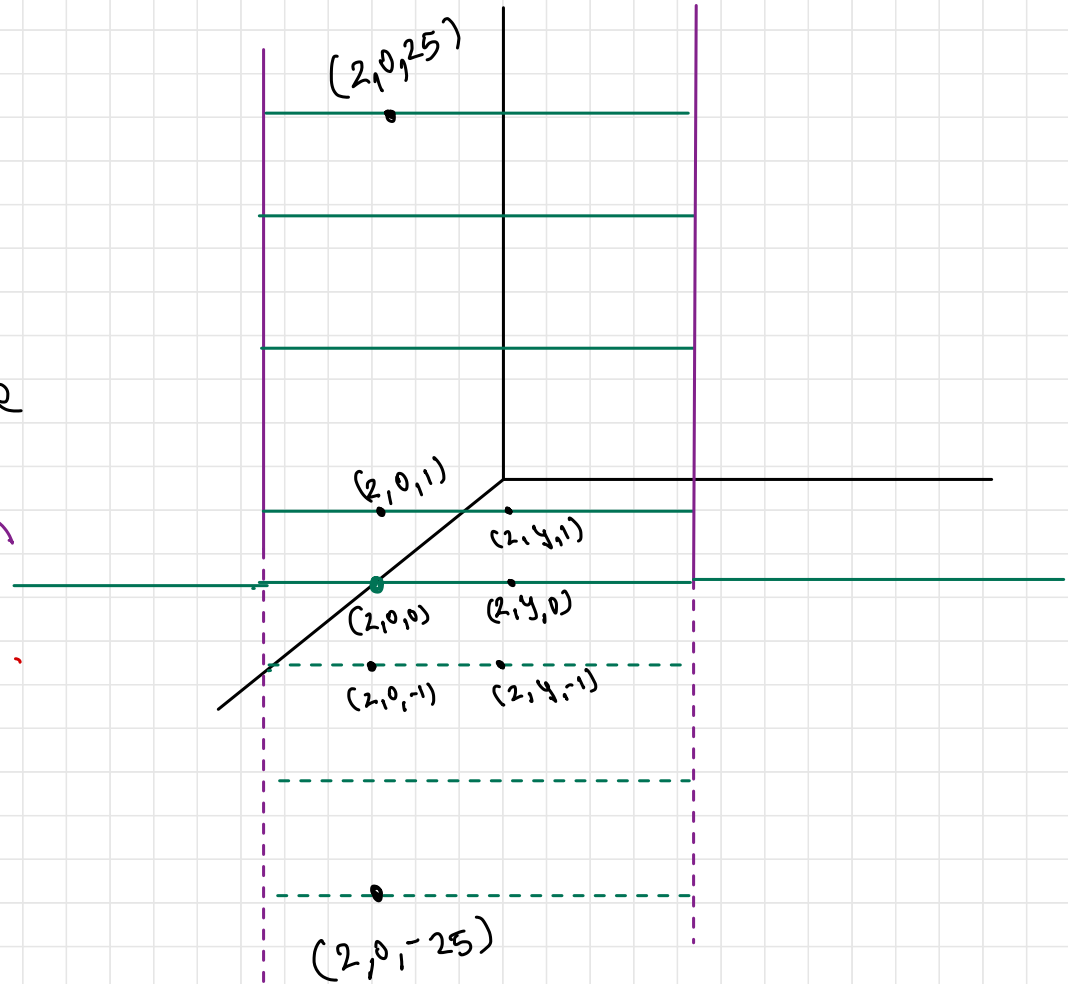
y is free

z is free

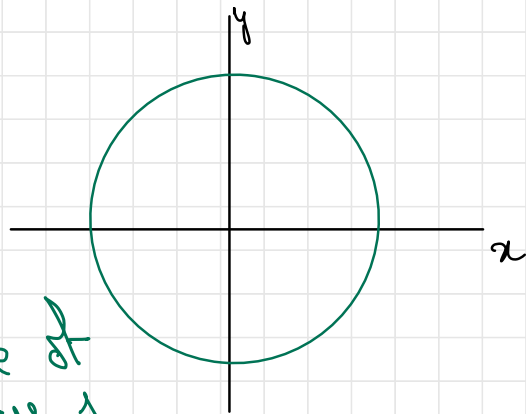
Stack all the
lines

in z -direction

Get a plane.

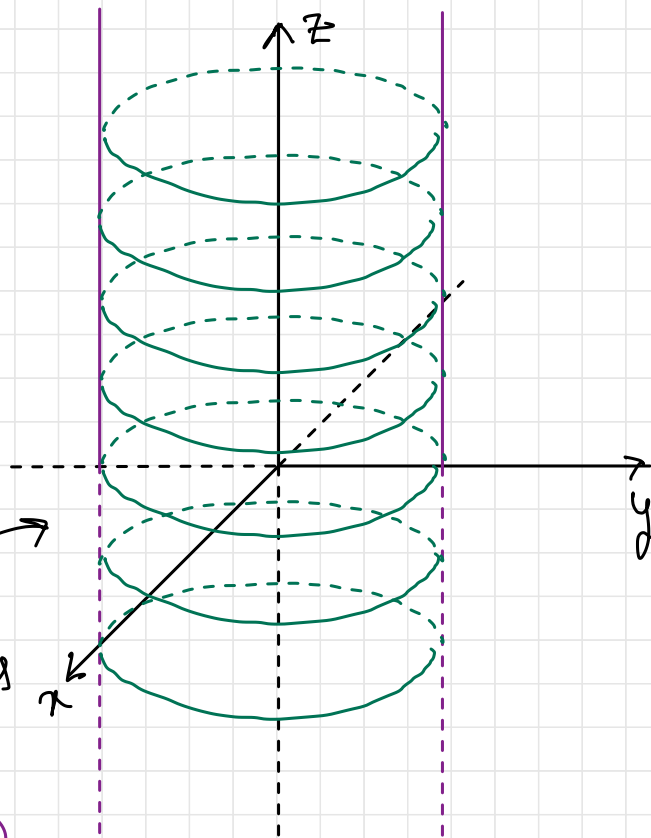


$$x^2 + y^2 = 1 \quad \leadsto \quad z \text{ is a Free Variable}$$



Circle of
Radius 1
in
xy plane ($z=0$)

Stack of Circles
of
Radius 1 in
z-direction



Circular Cylinder Parallel to z-axis

Traces: Slices / Curves / Cross-sections
that we stack to get 3D picture

$z=0 \rightarrow xy$ trace $\rightarrow$ intersection of the surface with xy plane

$z=k$ trace $\rightarrow$ intersection of surface with

$z=k$ plane $\rightarrow$ plane parallel to xy plane

$x=k$ trace $\rightarrow$ intersection of surface with

$x=k$ plane $\rightarrow$ plane parallel to yz plane

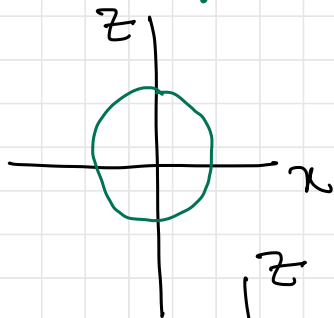
$y=k$ trace $\rightarrow$ intersection of surface with

$y=k$ plane $\rightarrow$ plane parallel to xz plane

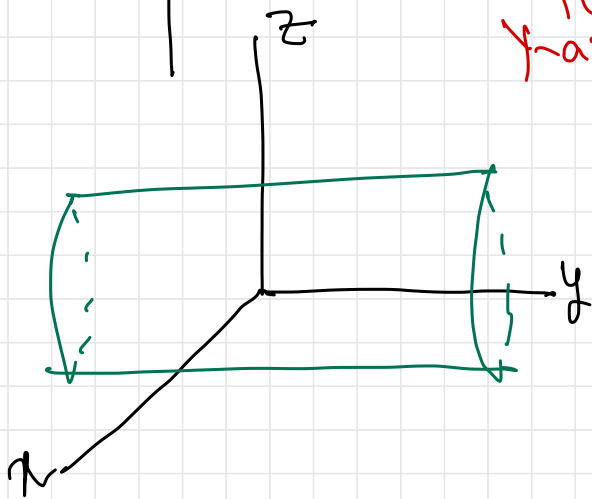
$$x^2 + z^2 = 1$$

y is free

→ stack traces in y -direction

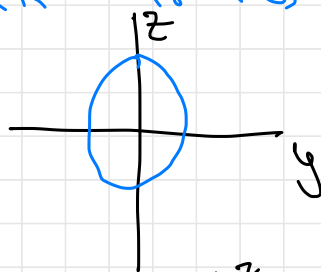


Circular
Cylinder
parallel
to
 y -axis

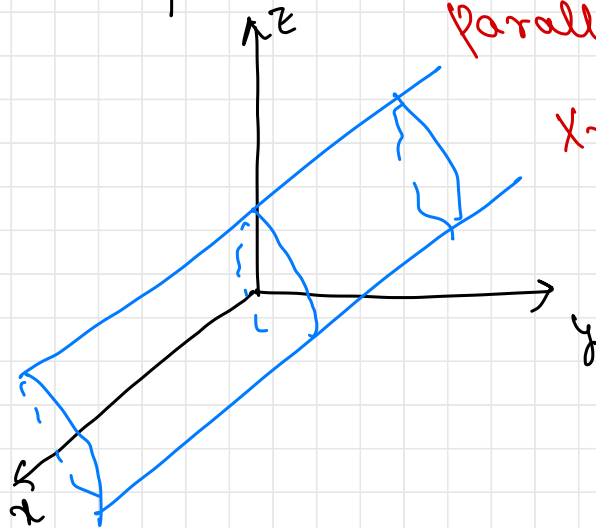


$$y^2 + z^2 = 1$$

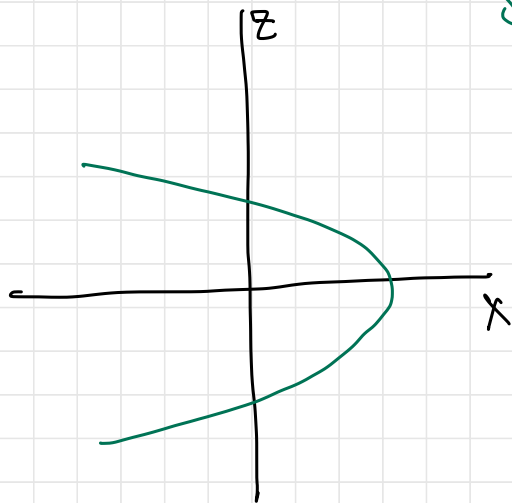
x is free
stack traces in x -direction



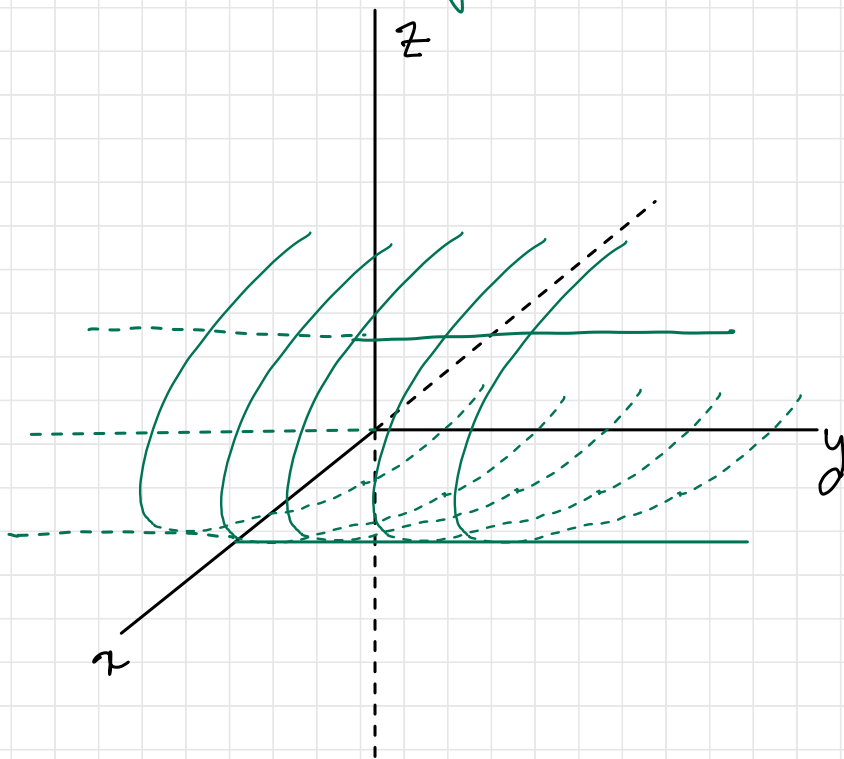
Circular
Cylinder
parallel
to
 x -axis



2. $x = 7 - z^2 \rightarrow y$ is free
 stack traces in y -direction



parabolic
 cylinder
 parallel to
 y -axis



Break

Stand up and Stretch

Reflect

Ask Questions

talk to your neighbor

.....

Next: Sphere, Cone and paraboloid

$$x^2 + y^2 + z^2 = 25$$

traces in z -direction:

$$x^2 + y^2 = 25 - z^2$$

$$z=0 \Rightarrow x^2 + y^2 = 25$$

$$\left. \begin{matrix} z=1 \\ z=-1 \end{matrix} \right\} \Rightarrow x^2 + y^2 = 24$$

$$\left. \begin{matrix} z=2 \\ z=-2 \end{matrix} \right\} x^2 + y^2 = 21$$

$$\left. \begin{matrix} z=4 \\ z=-4 \end{matrix} \right\} x^2 + y^2 = 9$$

$$\left. \begin{matrix} z=5 \\ z=-5 \end{matrix} \right\} x^2 + y^2 = 0 \Rightarrow \begin{matrix} (0,0,5) \\ (0,0,-5) \end{matrix}$$

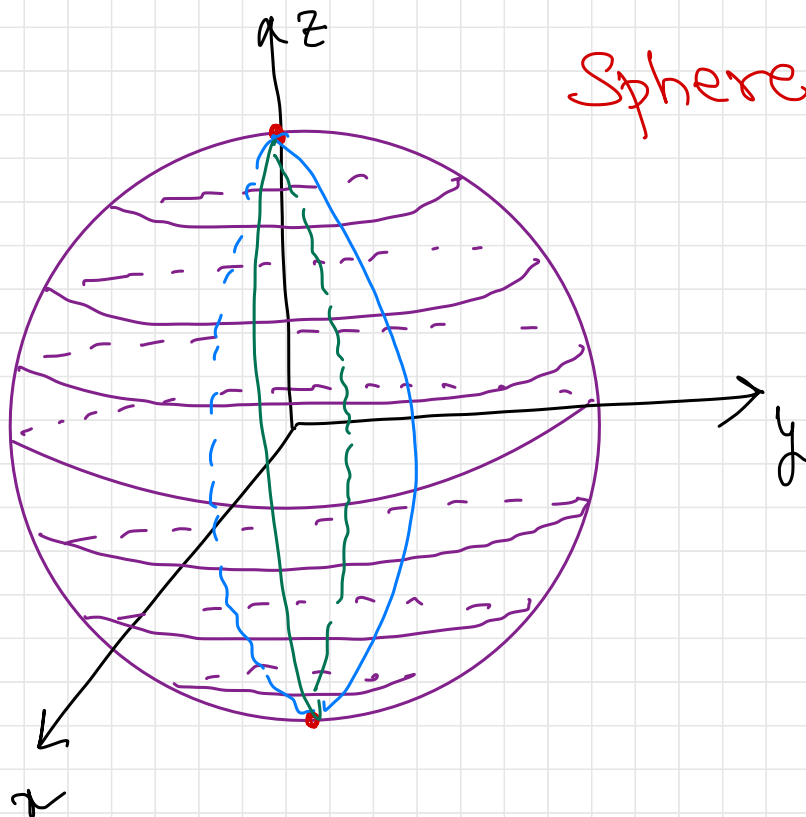
$$\left. \begin{matrix} z=5.1 \\ z=-5.1 \end{matrix} \right\} x^2 + y^2 = 5^2 - (5.1)^2 < 0$$

No traces Above 5 or below -5

$$x=0 \Rightarrow yz \text{ trace} \Rightarrow y^2 + z^2 = 25$$

$$y=0 \Rightarrow xz \text{ trace} \Rightarrow x^2 + z^2 = 25$$

Sphere



$$x^2 + y^2 = z^2$$

traces in z -direction

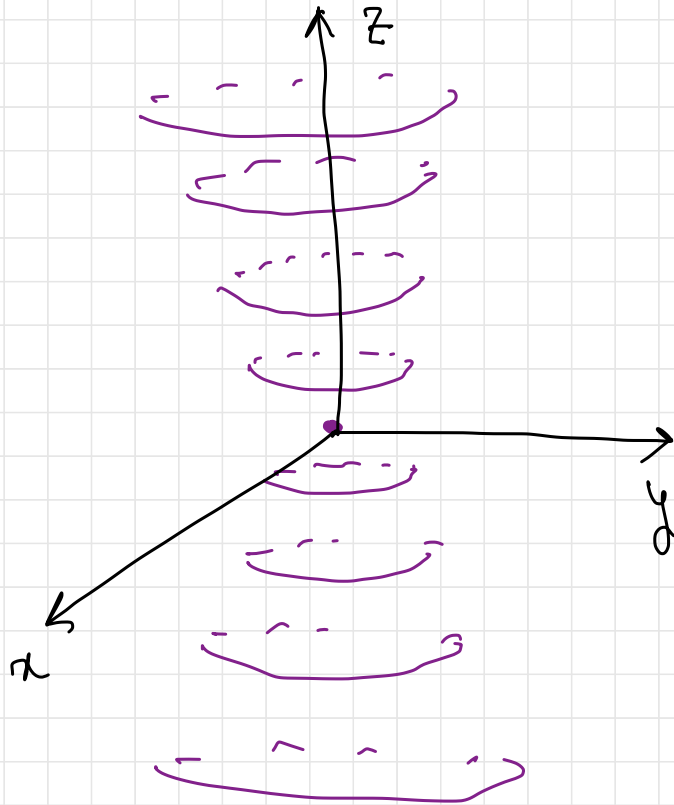
$$z=0 \leadsto x^2 + y^2 = 0$$

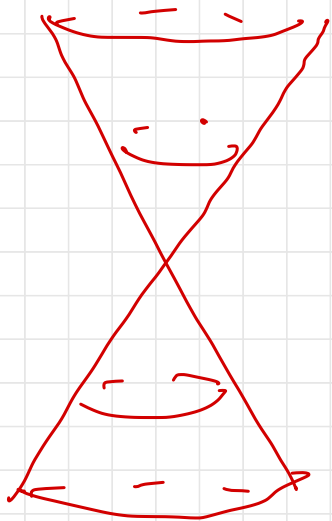
$$\left. \begin{array}{l} z=1 \\ z=-1 \end{array} \right\} x^2 + y^2 = 1$$

$$\left. \begin{array}{l} z=2 \\ z=-2 \end{array} \right\} x^2 + y^2 = 4$$

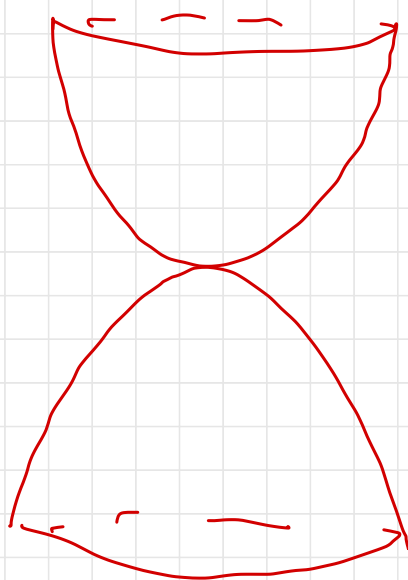
$$\left. \begin{array}{l} z=3 \\ z=-3 \end{array} \right\} x^2 + y^2 = 9$$

$$\left. \begin{array}{l} z=4 \\ z=-4 \end{array} \right\} x^2 + y^2 = 16$$





or
→

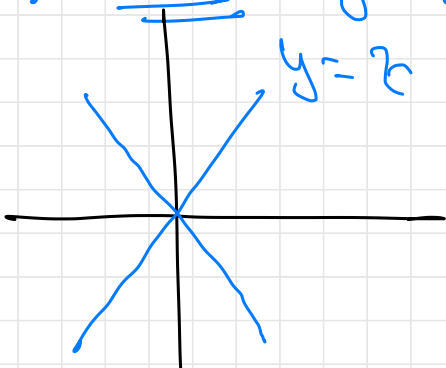


?

Find traces in other

$x=0$ $\leadsto y^2 = z^2$

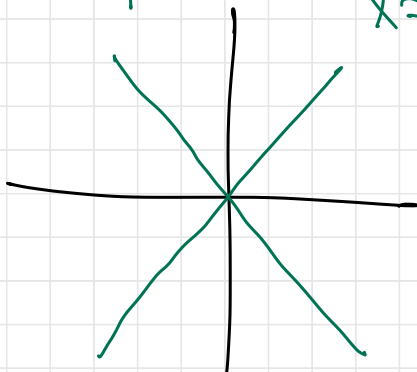
$y = z$ or $y = -z$



Directions

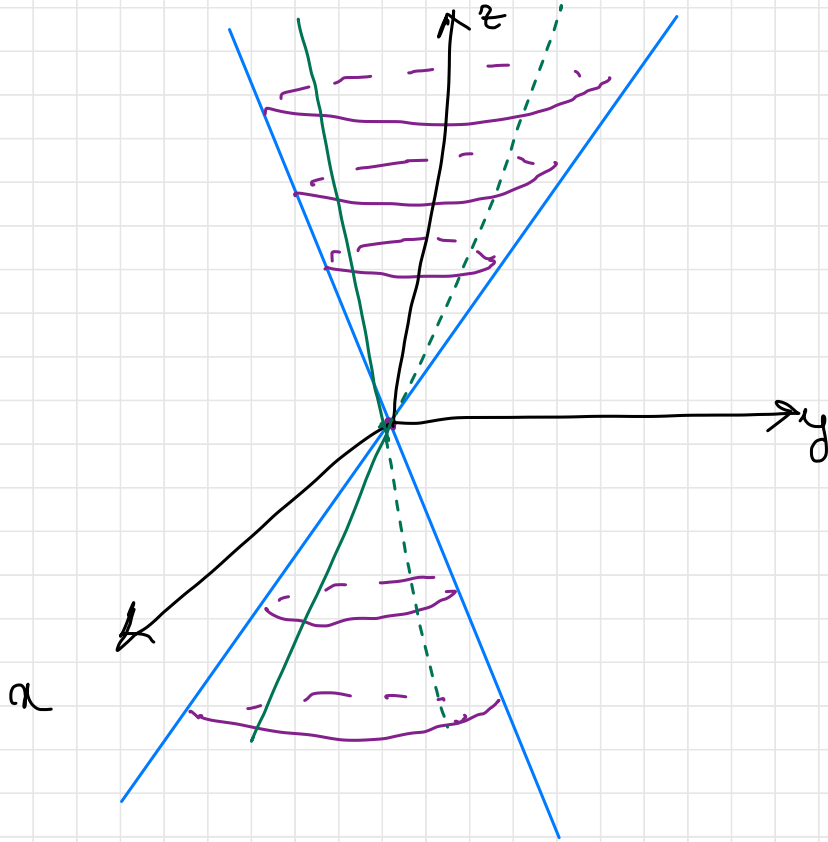
$y=0 \leadsto$

$x^2 = z^2$
 $x = z$ or $x = -z$



Double Cone:

$$x^2 + y^2 = z^2$$



eg: $z = x^2 + y^2$

No traces for $z < 0$

$z=0 \rightarrow x^2 + y^2 = 0$

$z=1 \rightarrow x^2 + y^2 = 1$

$z=2 \rightarrow x^2 + y^2 = 2$

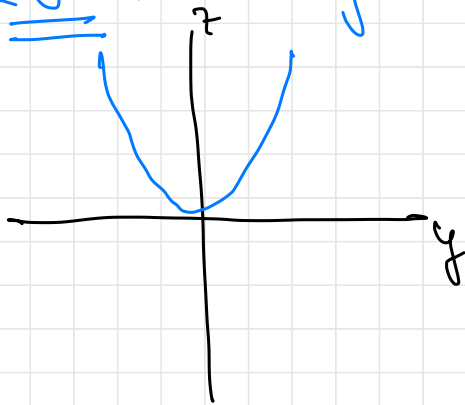
$z=4 \rightarrow x^2 + y^2 = 4$

Circles of
increasing

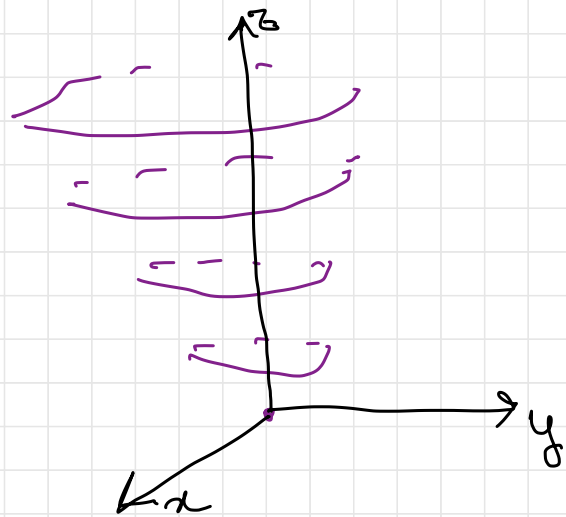
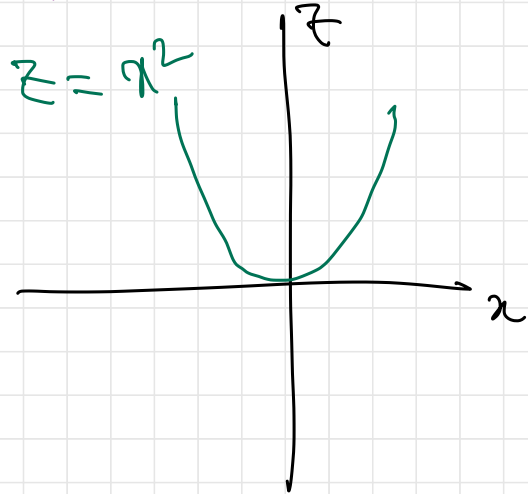
Radius
as

we go up in
 z -direction

$x=0 \rightarrow z=y^2$



$y=0 \rightarrow z=x^2$



$z = x^2 + y^2$ is a Paraboloid

