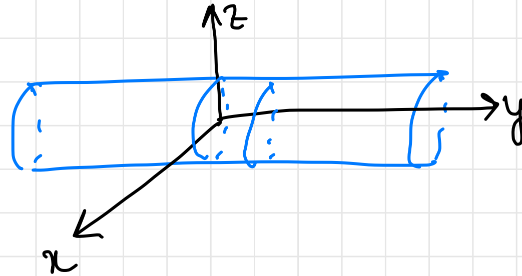
Lesson 4: Cylinders and Quadric Surfaces (13.6) - Part II

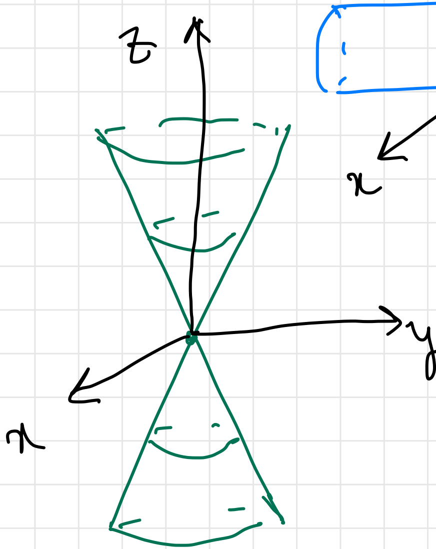
Warm-up: Recall sketches from last class

① $x^2 + z^2 = 1$
"y is free"



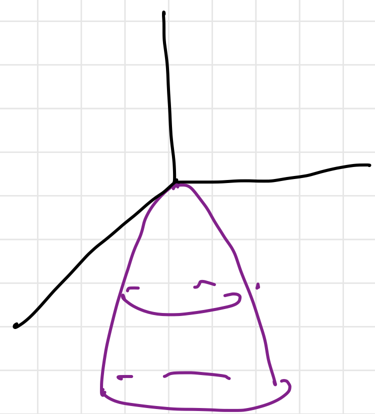
Circular Cylinder parallel to y-axis

② $x^2 + y^2 = z^2$



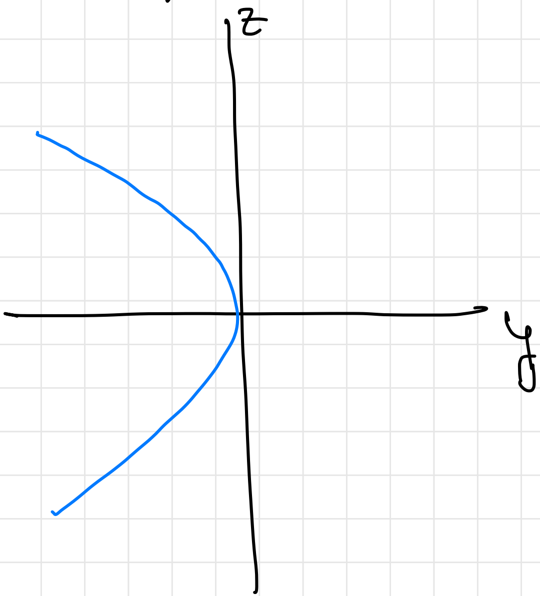
Circular Cone

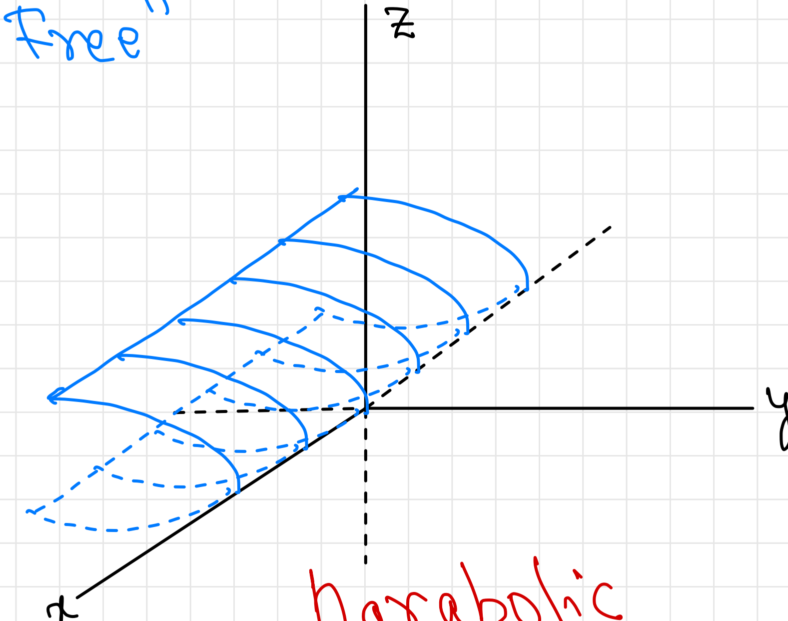
③ $x^2 + y^2 = -z$
No traces for $z > 0$
$z < 0$ is circles



paraboloid

eg: $y = -z^2$ "x is free"



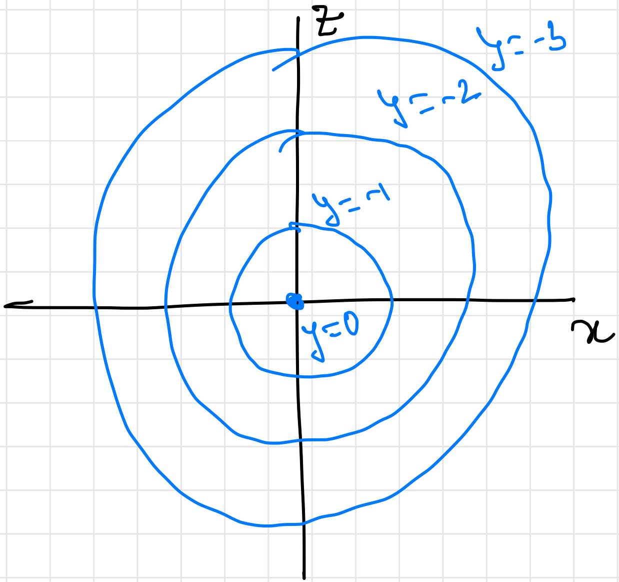


parabolic cylinder.

eg: $x^2+z^2=-y$

$(x^2+y^2=-z)$

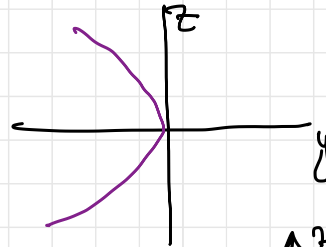
traces in y- direction:



$y>0$ No traces

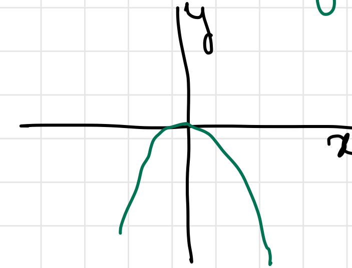
x- direction

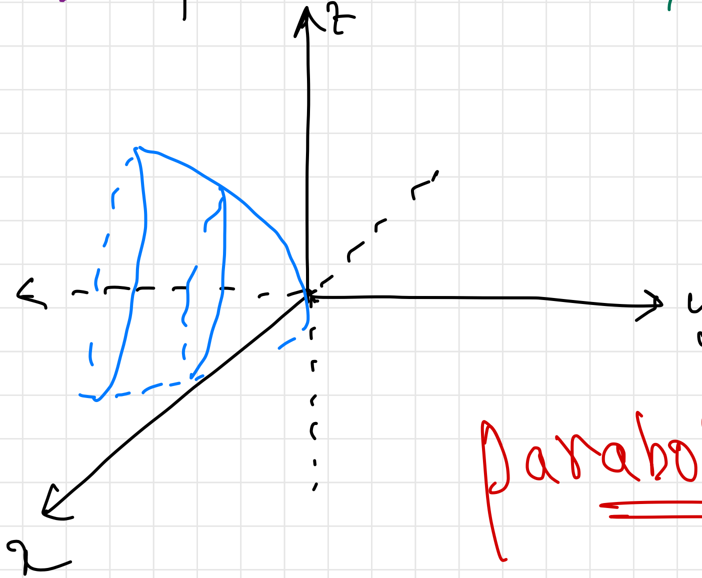
$x=0$ $z^2=-y$



z- direction

$z=0$ $x^2=-y$





Paraboloid

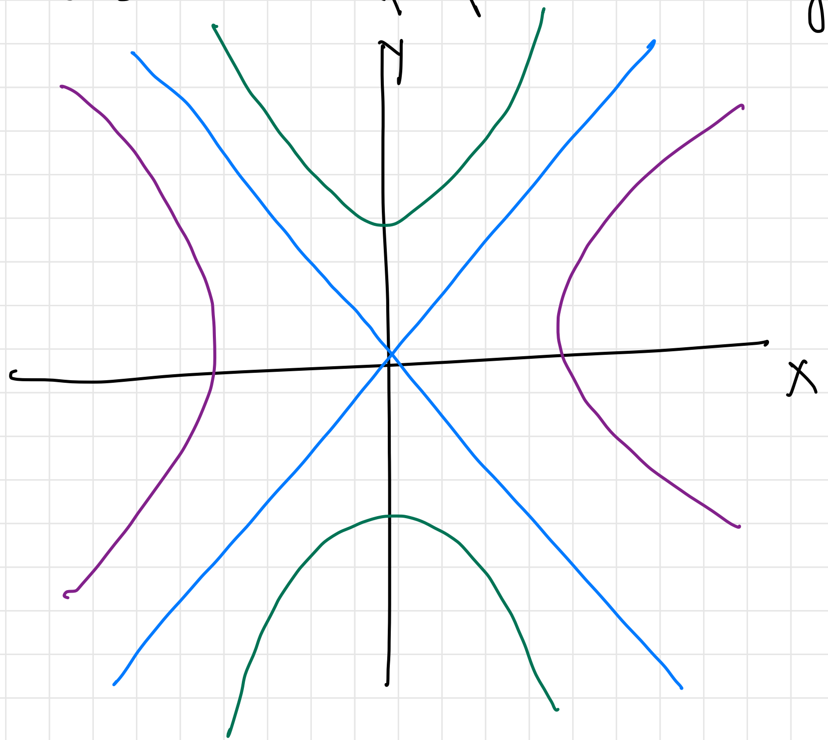
Think - Pair - Share

in $\mathbb{R}^2$ what does the graph of $x^2 - y^2 = k$ Represent?

$k=0$

$k>0$

$k<0$



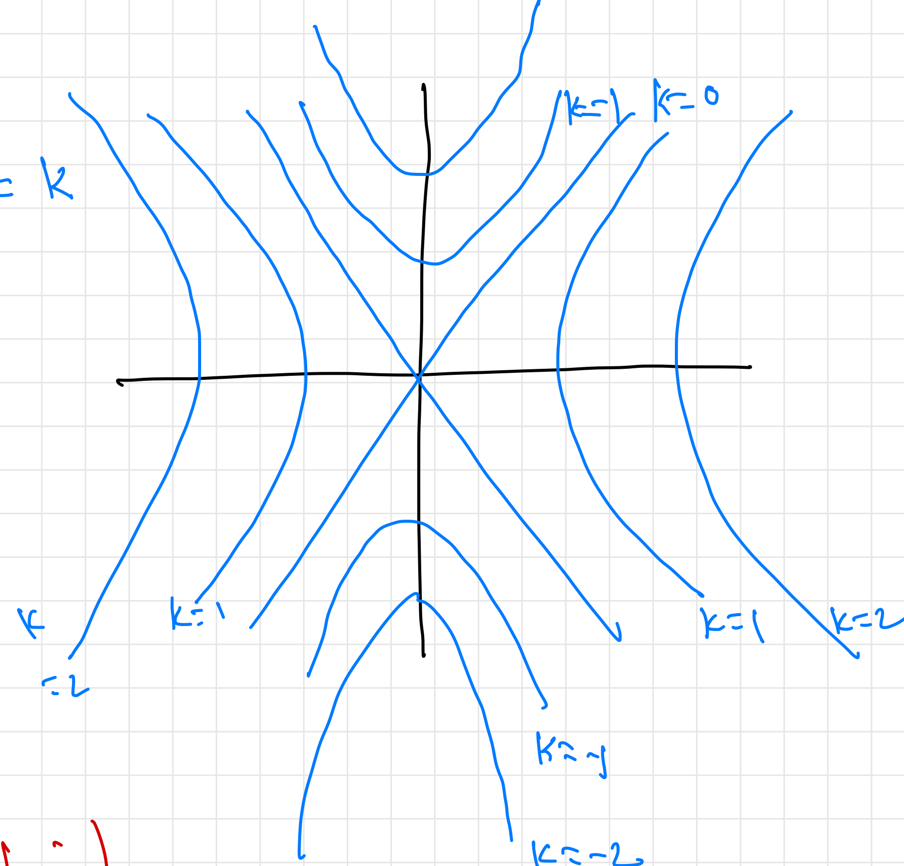
eg: $z = x^2 - y^2$

z-direction traces:

$z = k \rightsquigarrow x^2 - y^2 = k$

$k = 1 \rightsquigarrow x^2 - y^2 = 1$

$k = 2 \rightsquigarrow x^2 - y^2 = 2$



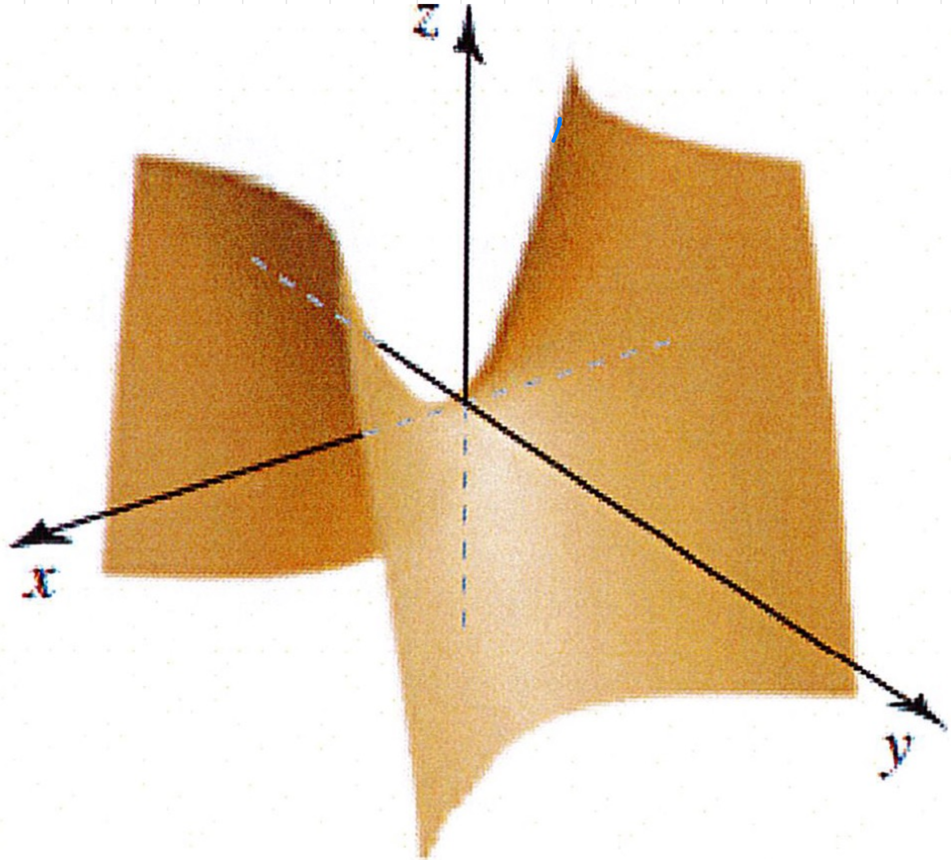
x-direction:

parabolas

y-direction:

parabolas

Hyperbolic Paraboloid


z
x
y

Break!!

Stretch

Walk around

Ask Questions

Reflect

Talk to your neighbors

– – . . . . . .

eg: $4x^2+y^2-z^2=36$

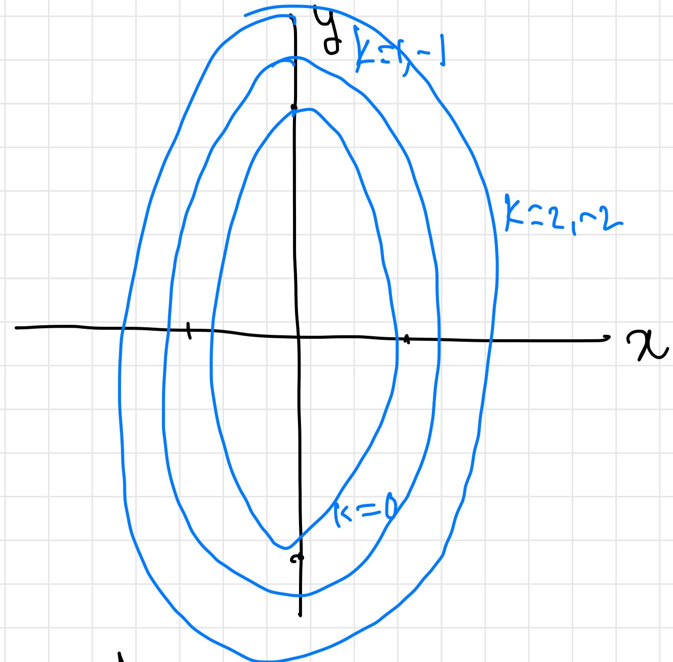
traces in z-direction:

$z=k \rightsquigarrow 4x^2+y^2=36+k^2$

$k=0 \rightsquigarrow 4x^2+y^2=36$

$k=1,-1 \rightsquigarrow 4x^2+y^2=37$

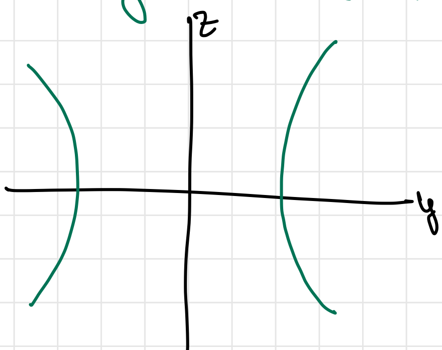
$k=2,-2 \rightsquigarrow 4x^2+y^2=40$



x-direction:

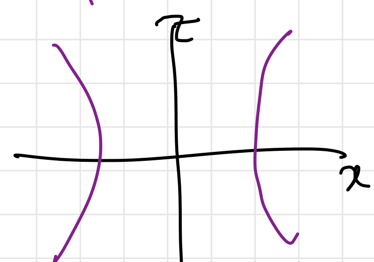
$x=k \rightsquigarrow y^2-z^2=36-4k^2$

Hyperbola

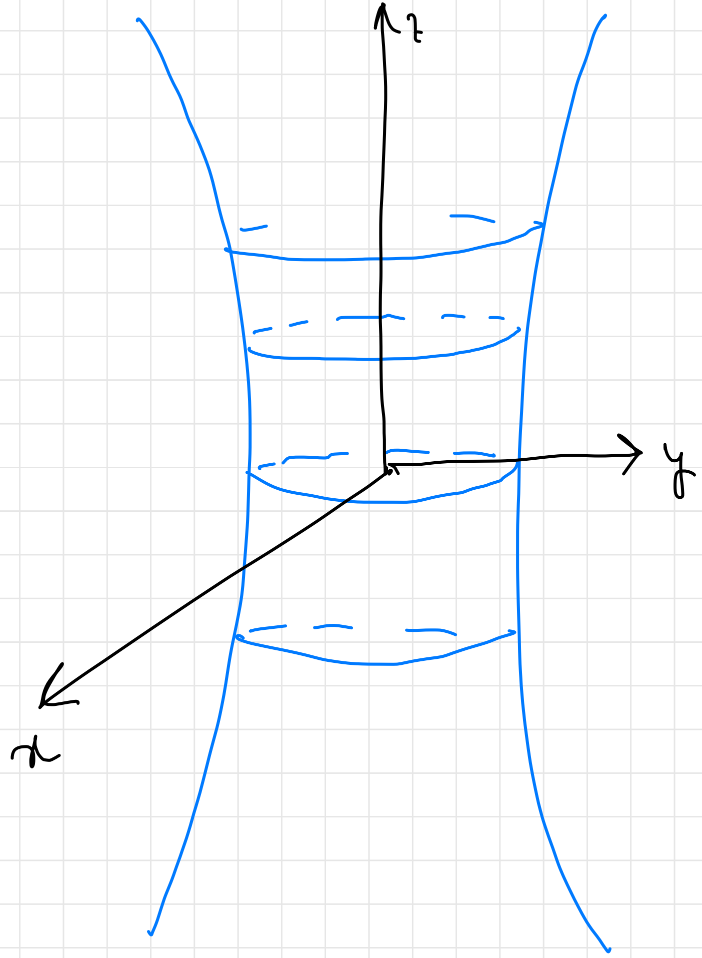


y-direction:

$y=k \rightsquigarrow 4x^2-z^2=36-k^2$



Hyperbola



Hyperboloid of 1 sheet

eg: $-x^2-y^2+4z^2=36$

z-direction traces)

$z=k \Rightarrow x^2+y^2=4k^2-36$

No traces when $4k^2-36<0$

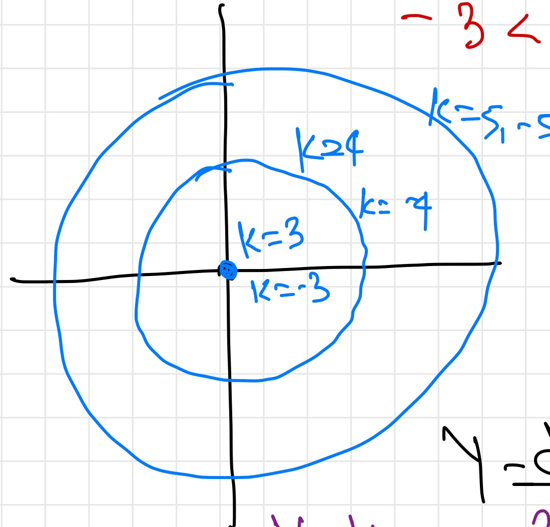
$k^2<9$

$-3<k<3$

$k=3,-3 \Rightarrow x^2+y^2=0$

$k=4,-4 \Rightarrow x^2+y^2=28$

$k=5,-5 \quad x^2+y^2=64$



X-direction

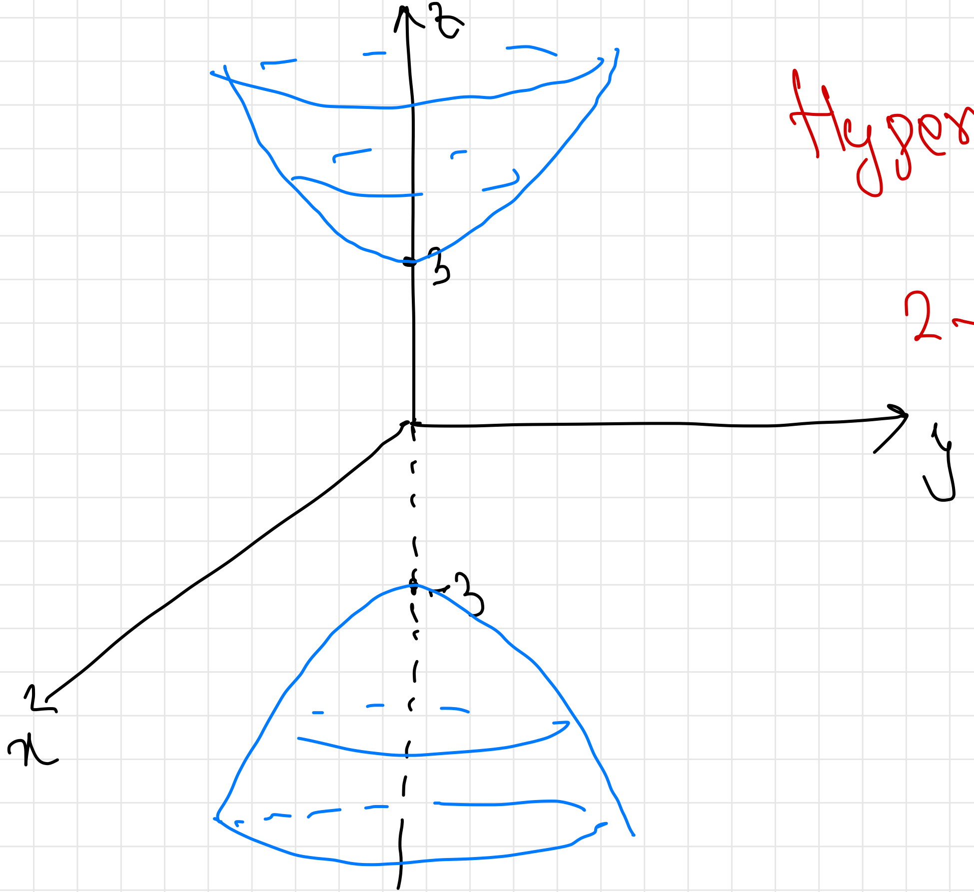
$X=k \quad -y^2+4z^2=36+k^2$

Hyperbola

Y-direction)

$Y=k \quad -x^2+4z^2=36+k^2$

Hyperbola



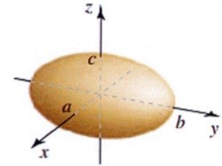
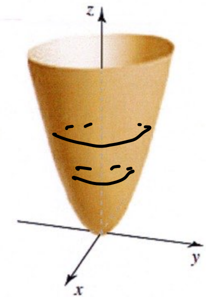
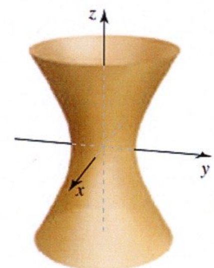
Hyperboloid of 2-sheets

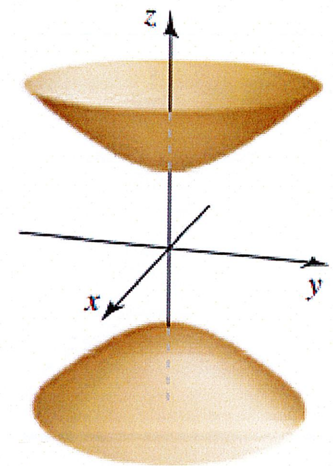
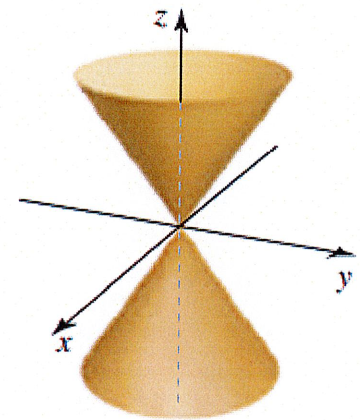
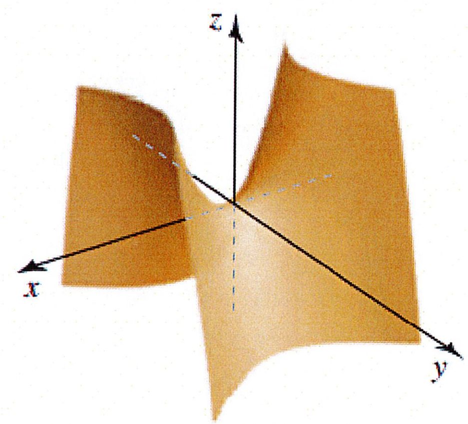
General form of Quadric Surfaces: → Degree ≤ 2 Powers

$$Ax^2+By^2+Cz^2+Dxy+Eyz+Fxz+Gx+Hy+Iz+J=0$$

Traces are $\underbrace{\text{Quadric Curves}}$ in all Directions

- lines
- circles
- Ellipses
- parabolas
- Hyperbolas

Name	Standard Equation	Features	Graph
Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ $a=b=c \rightsquigarrow$ sphere	All traces are ellipses.	
Elliptic paraboloid	$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$ $a=b \rightsquigarrow$ parabolid	Traces with $z=z_0>0$ are ellipses. Traces with $x=x_0$ or $y=y_0$ are parabolas.	
Hyperboloid of one sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	Traces with $z=z_0$ are ellipses for all z_0. Traces with $x=x_0$ or $y=y_0$ are hyperbolas.	

Hyperboloid of two sheets	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Traces with $z=z_0$ with $\lvert z_0\rvert > \lvert c\rvert$ are ellipses. Traces with $x=x_0$ and $y=y_0$ are hyperbolas.	
Elliptic cone	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$	Traces with $z=z_0\neq 0$ are ellipses. Traces with $x=x_0$ or $y=y_0$ are hyperbolas or intersecting lines.	
Hyperbolic paraboloid	$z=\frac{x^2}{a^2}-\frac{y^2}{b^2}$	Traces with $z=z_0\neq 0$ are hyperbolas. Traces with $x=x_0$ or $y=y_0$ are parabolas.	

$a=b \rightsquigarrow$ Circular Cone