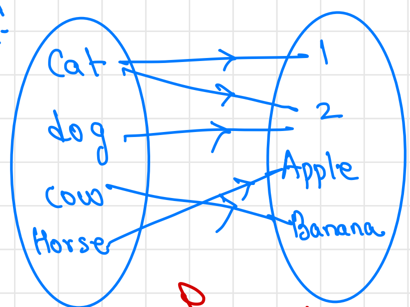
Lesson 5: Vector Valued Functions (14.1)

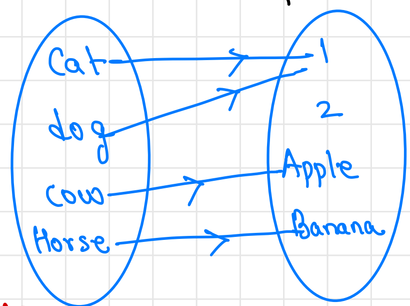
Warm-up: ① What is a function? { One output for each input

② What are some examples of functions?

eg:



NOT Function



Function

Domain = All possible inputs

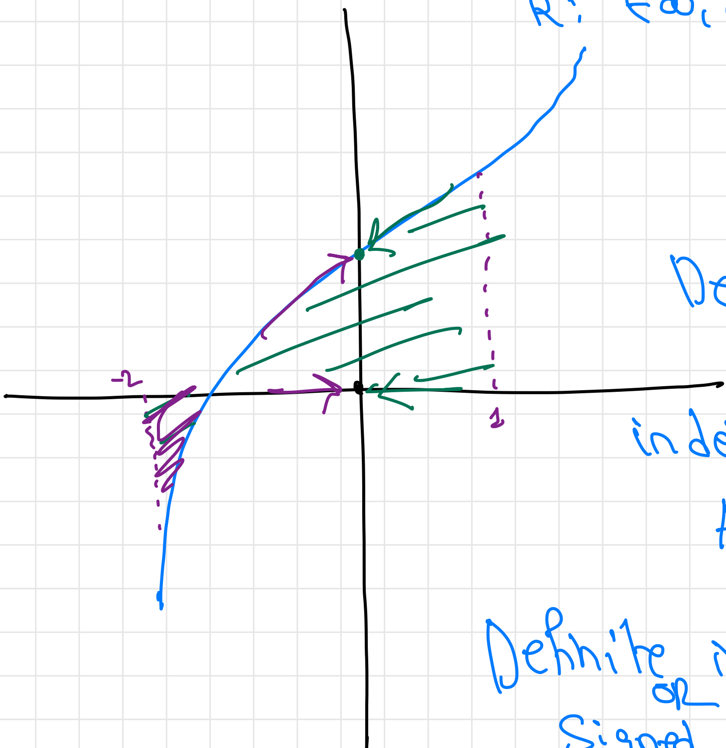
Range = all outputs

eg: $f(x) = x^2$ { D: $(-\infty, \infty)$, R: $[0, \infty)$

eg: $f(x) = \ln x$ { D: $(0, \infty)$, R: $(-\infty, \infty)$

Recall: Scalar Functions $\begin{cases} \text{input: Number} \\ \text{Output: Number} \end{cases}$

$f(x) = x^3 + 1$, D: $(-\infty, \infty)$
R: $(-\infty, \infty)$



limits: $\lim_{x \to 0} f(x) = \underbrace{1 = f(0)}$

Continuous at $x = 0$

Derivative: $\frac{d}{dx}(x^3+1) = 3x^2$

indefinite integral OR Antiderivative: $\int x^3 + 1 \, dx = \underbrace{\frac{x^4}{4} + x + C}_{F(x)}$

Definite integral OR Signed Area: $\int_{-2}^{1} f(x) \, dx = F(1) - F(-2)$ } FTC

Vector Valued Functions

input = Number
Output = Vector

Notation! $\vec{r}(t) = \langle f(t), \; g(t), \; h(t) \rangle$

each of $f(t)$, $g(t)$, $h(t)$ are scalar function

Domain of $\vec{r}(t)$ = intersection of Domains of $f(t)$, $g(t)$, $h(t)$

eg: $\vec{r}(t) = \langle t^3+5, \ln(3-t), \sqrt{t} \rangle$, find Domain

Domain:

$(-\infty, \infty)$

Domain:

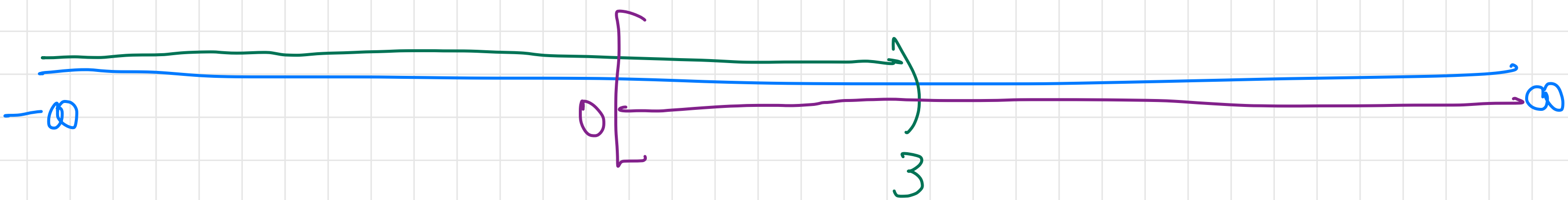
$3-t > 0$

$t < 3$

$(-\infty, 3)$

Domain:

$[0, \infty)$



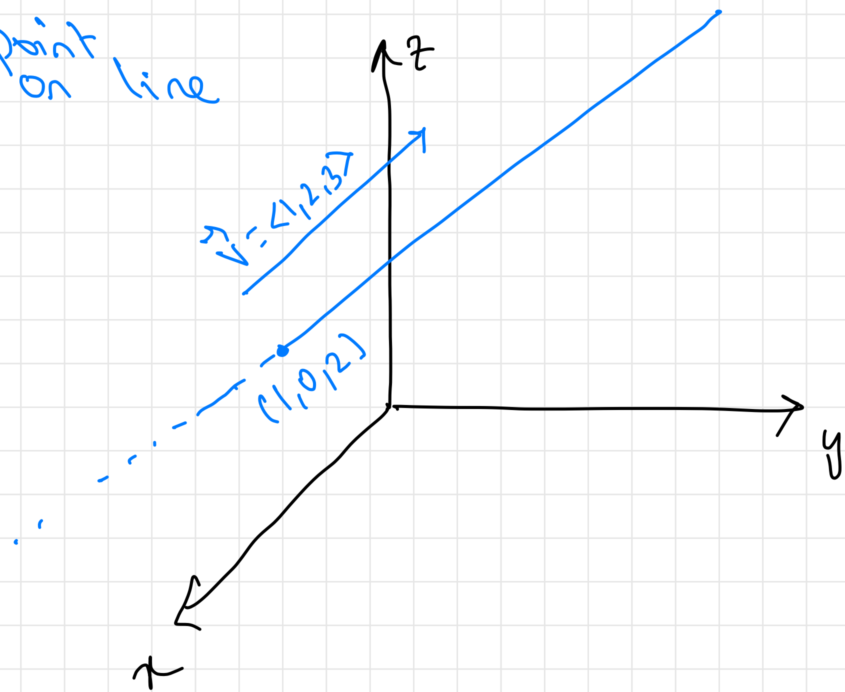
Domain of $\vec{r}(t)$ is $[0, 3)$

eg: $\vec{r}(t) = \langle 1+t, 2t, 2+3t \rangle \rightsquigarrow$ Domain: $(-\infty, \infty)$

$= \langle 1, 0, 2 \rangle + t\langle 1, 2, 3 \rangle$

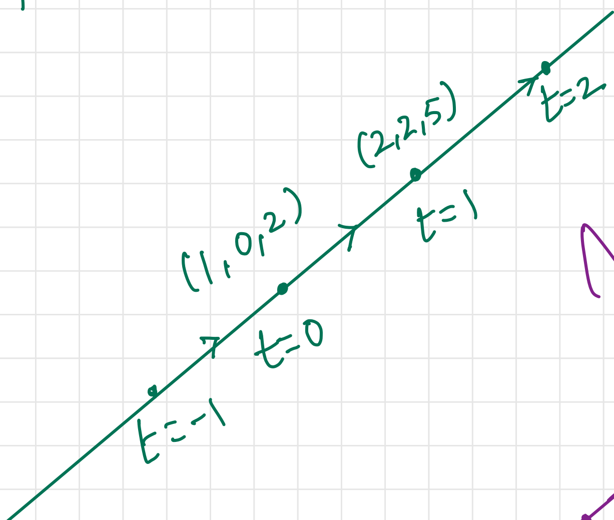
Line $\vec{r}(t) = \vec{r}_0 + t\vec{v}$ → Direction vector

One point on line

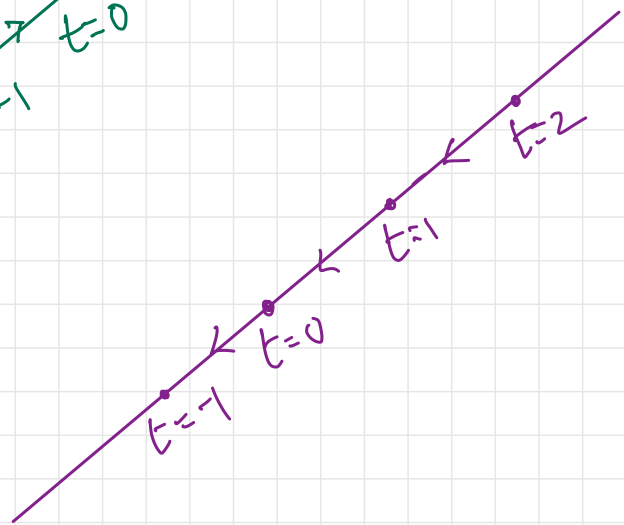


Orientation

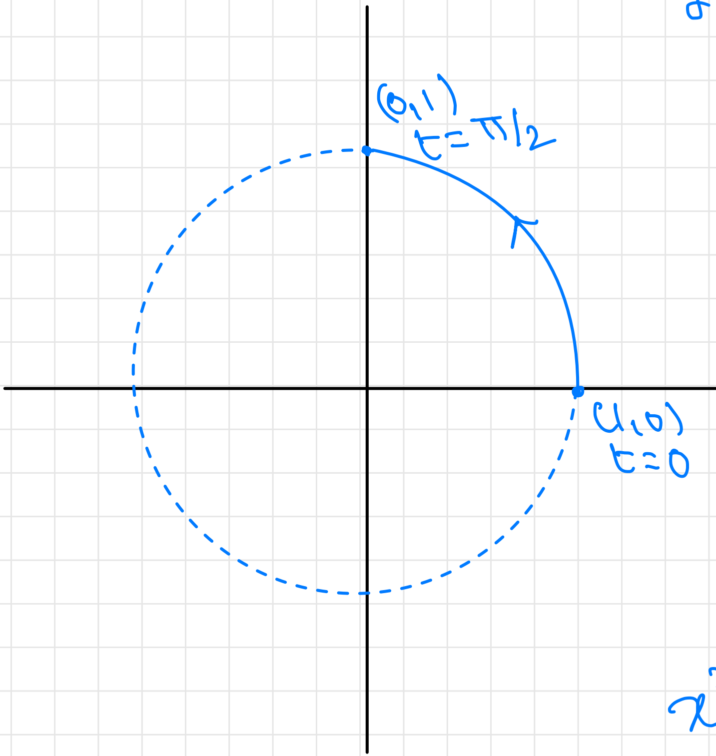
positive orientation: Go from lower t values to higher t values



Negative orientation: Go from higher t values to lower t- values



eg: $\vec{r}(t) = \langle \underbrace{\cos t}_{x}, \underbrace{\sin t}_{y} \rangle, \quad 0 \le t \le \pi/2$

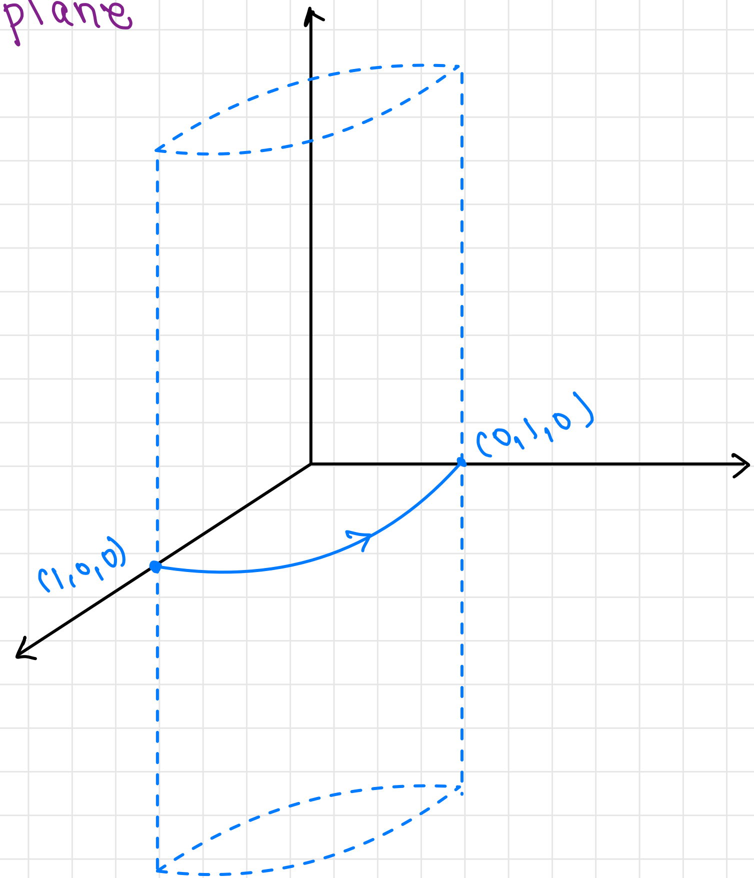


t	x	y
0	1	0
$\pi/6$	$\sqrt{3}/2$	$1/2$
$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\pi/3$	$1/2$	$\sqrt{3}/2$
$\pi/2$	0	1

$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

eg: $\vec{r}(t) = \langle \underbrace{\cos t, \sin t}_{x^2+y^2=1}, \underset{\substack{z=0 \\ xy \text{ plane}}}{0} \rangle, \quad 0 \le t \le \pi/2$

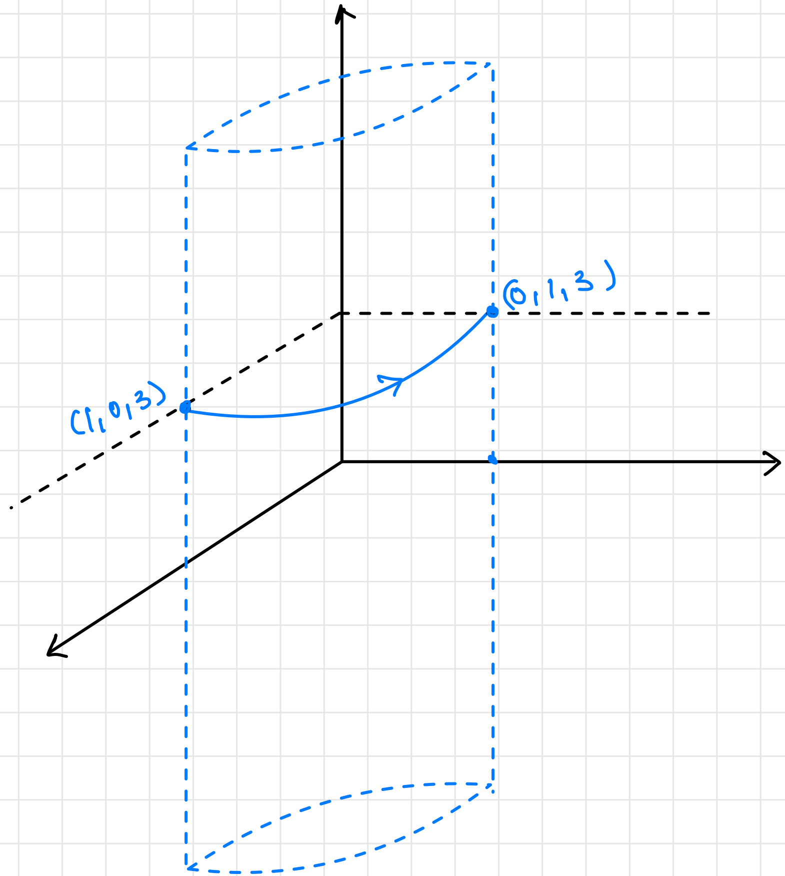
Cylinder in 3D



eg:

$\vec{r}(t) = \langle \cos t, \sin t, 3 \rangle, \quad 0 \le t \le \pi/2$

$z = 3$



Break!!

Stretch

Walk around

Ask Questions

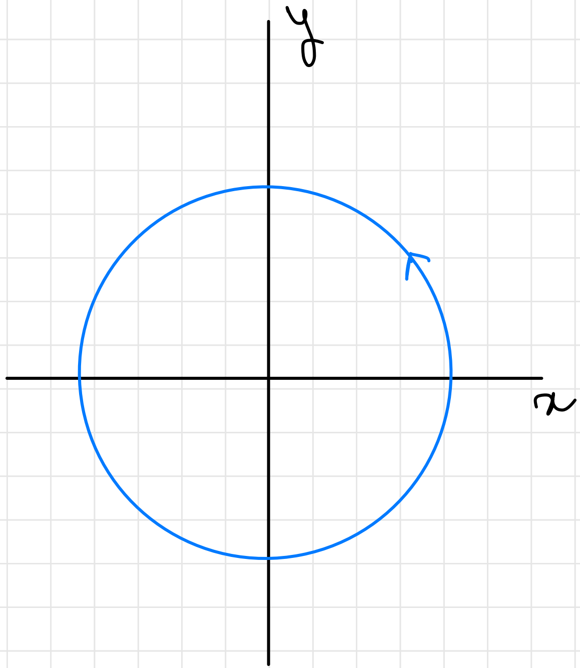
Reflect

Talk to your neighbors

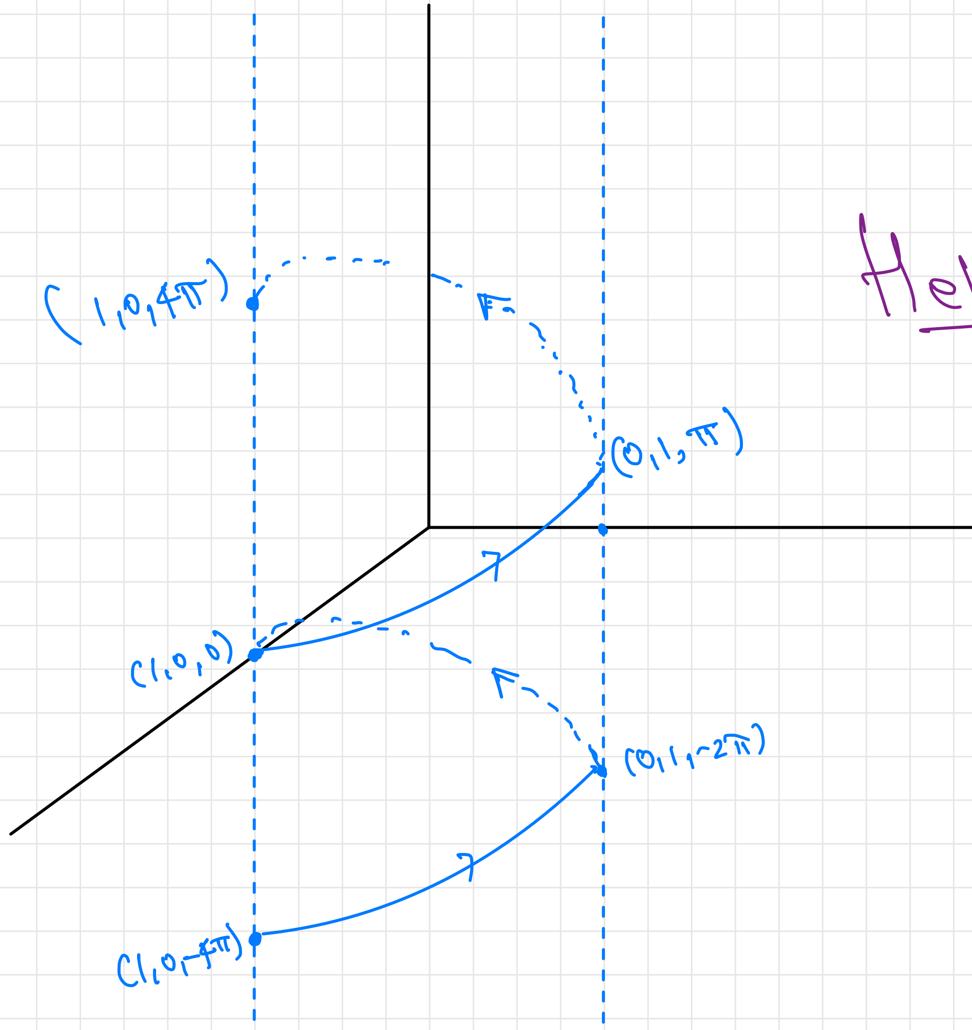
— — . . . . . ^ ^

eg: $\vec{r}(t) = \langle \cos t, \sin t, 2t \rangle$, $-2\pi \le t \le 2\pi$

$x^2 + y^2 = 1$



t	x	y	z
-2π	1	0	-4π
$-3\pi/2$	0	1	-3π
$-\pi$	-1	0	-2π
$-\pi/2$	0	-1	$-\pi$
0	1	0	0
$\pi/2$	0	1	π
π	-1	0	2π
$3\pi/2$	0	-1	3π
2π	1	0	4π



Helix

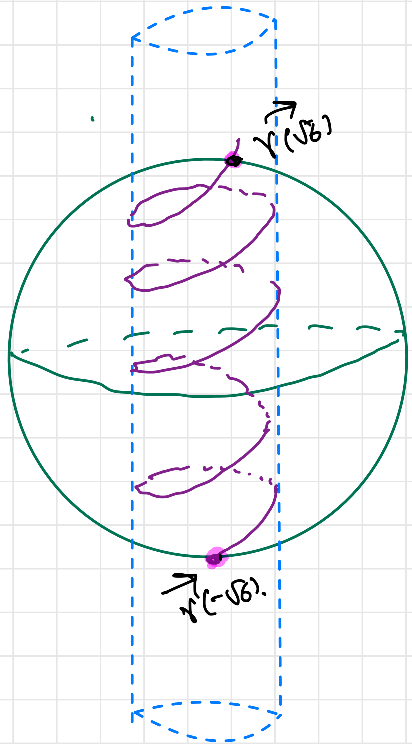
intersection with a Surface

Does $\vec{r}(t) = \langle \cos t, \sin t, 2t \rangle$, $-\infty < t < \infty$

Helix on Cylinder $x^2+y^2=1$

intersect with $x^2+y^2+z^2=25$

Sphere of Radius 5



intersects at two points!!

Finding points of intersection:

$$\vec{r}(t) = \langle \underset{x}{\cos t}, \underset{y}{\sin t}, \underset{z}{2t} \rangle$$

plug in x, y, z for $\vec{r}(t)$ into $x^2 + y^2 + z^2 = 25$

$$\underbrace{\cos^2 t + \sin^2 t}_{1} + (2t)^2 = 25$$

$$4t^2 = 24$$

$$t^2 = 6$$

$$t = \sqrt{6}, -\sqrt{6}$$

Top

two points $\vec{r}(\sqrt{6}) = \langle \cos(\sqrt{6}), \sin(\sqrt{6}), 2\sqrt{6} \rangle$

$\vec{r}(-\sqrt{6}) = \langle \cos(-\sqrt{6}), \sin(-\sqrt{6}), -2\sqrt{6} \rangle$ bottom.