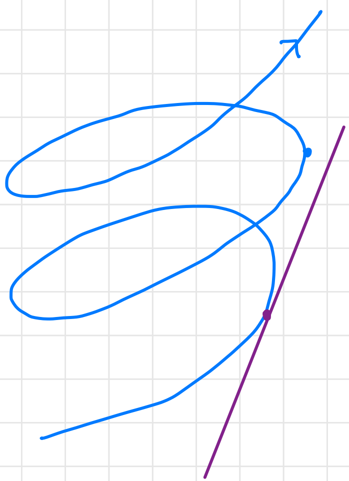
# Lesson 7: Motion in Space (14.3) - Part II

## Review example:

Given $\vec{r}(t) = \langle t\sin t, t\cos t, t \rangle$

what does $\vec{r}\,'(\pi/2)$ Represent?

$f(x) \rightsquigarrow f'(a)$ is slope of tangent line at $(a, f(a))$



$\vec{r}\,'(a) \rightsquigarrow$ represents direction vector of tangent line

$$\vec{r}\,'(t) = \langle \sin t + t\cos t, \cos t - t\sin t, 1 \rangle$$

$$\vec{r}\,'\left(\tfrac{\pi}{2}\right) = \left\langle 1, -\tfrac{\pi}{2}, 1 \right\rangle$$

Find equation of tangent line to

$\vec{r}(t) = \langle t\sin t, t\cos t, t\rangle$ at $(\pi/2, 0, \pi/2)$

$\vec{r}(\frac{\pi}{2}) = \langle \frac{\pi}{2}, 0, \frac{\pi}{2} \rangle$

$t\vec{v} + \vec{r}_0$

Direction vector

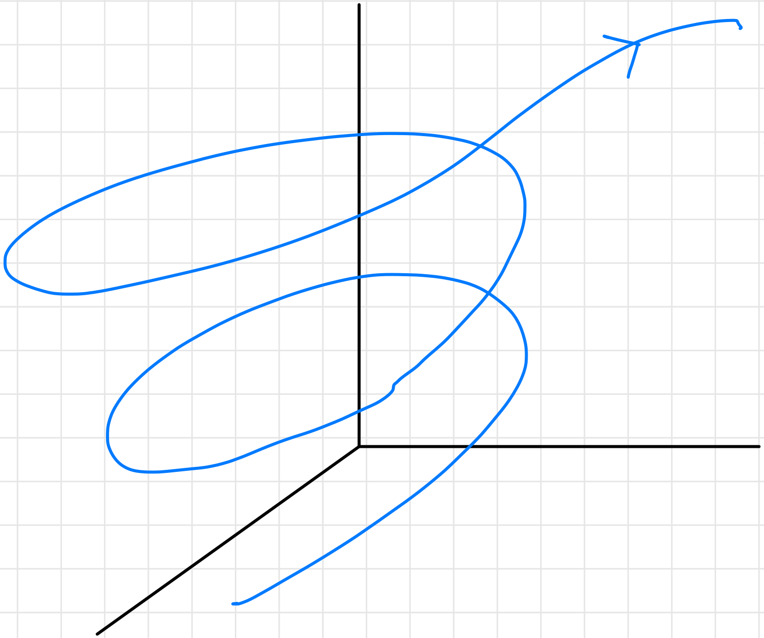
point on line

$t\,\vec{r}'(\frac{\pi}{2}) + r(\frac{\pi}{2})$

$$\boxed{t\langle 1, -\frac{\pi}{2}, 1\rangle + \langle \frac{\pi}{2}, 0, \frac{\pi}{2} \rangle}$$

equation of tangent line at $(\pi/2, 0, \pi/2)$

# Particle moving in 3D/2D



$\vec{r}(t) \rightsquigarrow$ position

$\vec{v}(t) = \vec{r}'(t) \rightsquigarrow$ velocity

$\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t) \rightsquigarrow$ Acceleration eg:

$$\vec{r}(t) = \langle e^{2t}, t^2, \sin(\pi t) \rangle \rightsquigarrow \text{position vector}$$

$$\vec{v}(t) = \vec{r}\,'(t) = \left\langle \frac{d}{dt} e^{2t}, \frac{d}{dt} t^2, \frac{d}{dt} \sin(\pi t) \right\rangle$$

$$= \langle 2e^{2t}, 2t, \pi\cos(\pi t) \rangle$$

$$\vec{v}(0) = \langle 2, 0, \pi \rangle$$

$$\vec{v}(1/2) = \langle 2e, 1, 0 \rangle$$

$$\vec{a}(t) = \vec{v}\,'(t) = \left\langle \frac{d}{dt}(2e^{2t}), \frac{d}{dt}(2t), \frac{d}{dt}(\pi\cos(\pi t)) \right\rangle$$

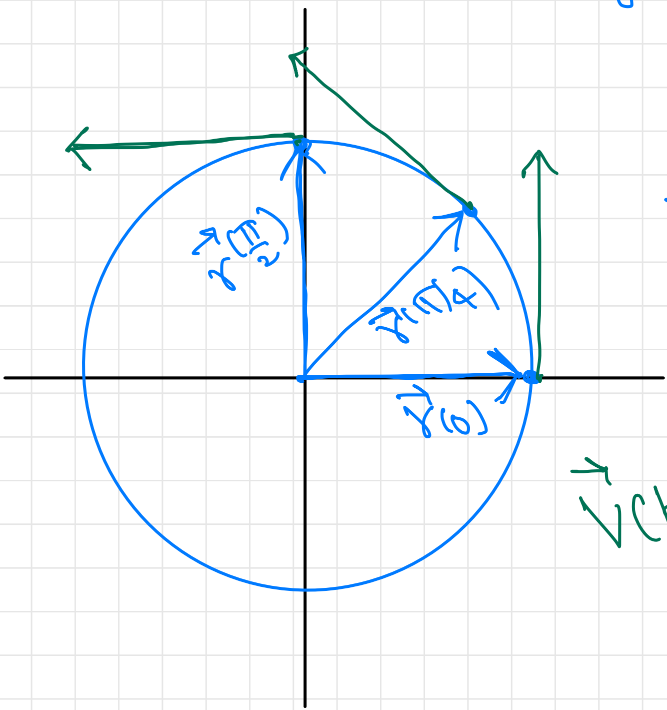
$$= \langle 4e^{2t}, 2, -\pi^2 \sin(\pi t) \rangle$$

$$\vec{a}(0) = \langle 4, 2, 0 \rangle$$

$$\vec{a}(1/2) = \langle 4e, 2, -\pi^2 \rangle$$

eg: $\vec{r}(t) = \langle \underbrace{\cos t}_{x(t)}, \underbrace{\sin t}_{y(t)} \rangle$, $0 \le t \le 2\pi$ (2D)

$\underbrace{x^2(t) + y^2(t) = \sin^2 t + \cos^2 t = 1}_{\text{Represents Circular Motion.}}$




$\vec{r}(0) = \langle 1, 0 \rangle$

$\vec{r}(\pi/4) = \langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$

$\vec{r}(\pi/2) = \langle 0, 1 \rangle$

$\vec{v}(t) = \langle -\sin t, \cos t \rangle$

$\vec{v}(0) = \langle 0, 1 \rangle$

$\vec{v}(\pi/4) = \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\vec{v}(\pi/2) = \langle -1, 0 \rangle$

$$\left.\begin{array}{l} \vec{r}(0) \cdot \vec{v}(0) = 0 \\ \vec{r}(\frac{\pi}{4}) \cdot \vec{v}(\frac{\pi}{4}) = 0 \\ \vec{r}(\frac{\pi}{2}) \cdot \vec{v}(\frac{\pi}{2}) = 0 \\ \vec{r}(t) \cdot \vec{v}(t) = 0 \end{array}\right.$$

* $\vec{v}(t) \perp \vec{r}(t)$ for all $t$.

# Circular Motion:

$\vec{r}(t) = \langle x(t), y(t) \rangle$ is in Circular Motion then

1) eliminate $t$, to write $x^2+y^2=R^2$

2) $\vec{r}(t) \perp \vec{v}(t)$ for all values of $t$

eg #1 $\vec{r}(t) = \langle \cos(t^2), \sin(t^2) \rangle$

$\rightsquigarrow \vec{v}(t) = \langle -2t\sin(t^2), 2t\cos(t^2) \rangle$

eg #1 $\vec{r}(t) = \langle \underbrace{R\cos(f(t))}_{x}, \underbrace{R\sin(f(t))}_{y} \rangle$

$\rightsquigarrow \vec{v}(t) = \langle -Rf'(t)\sin(f(t)), Rf'(t)\cos(f(t)) \rangle$

$x^2+y^2=R^2$, $\vec{r}(t) \perp \vec{v}(t)$

Think - pair - share (iclicker)

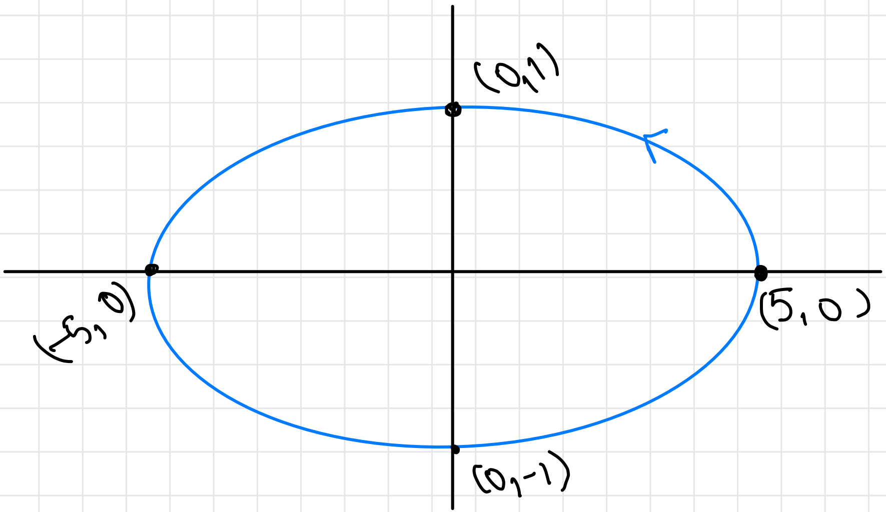
$\vec{r}(t) = \langle 5\cos t, \sin t \rangle$, $0 \le t \le 2\pi$.

Does this describe a circular motion?

$\vec{v}(t) = \langle -5\sin t, \cos t \rangle$

$\vec{r}(t) \cdot \vec{v}(t) = -25 \sin t \cos t + \sin t \cos t = -24 \sin t \cos t \neq 0$

NOT circular motion




$\frac{x}{5} = \cos t$, $y = \sin t$

$\frac{x^2}{25} + y^2 = 1$

$\rightsquigarrow$ Ellipse

# Review: Integration

constant vector of integration

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

then $$\int \vec{r}(t)\,dt = \left\langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt \right\rangle + \vec{C}$$

eg #: $$\vec{r}(t) = \langle t^2, e^t, \sqrt{t} \rangle$$

$$\int \vec{r}(t)\,dt = \left\langle \frac{t^3}{3}, e^t, \frac{2}{3}t^{3/2} \right\rangle + \vec{C}$$

indefinite integral

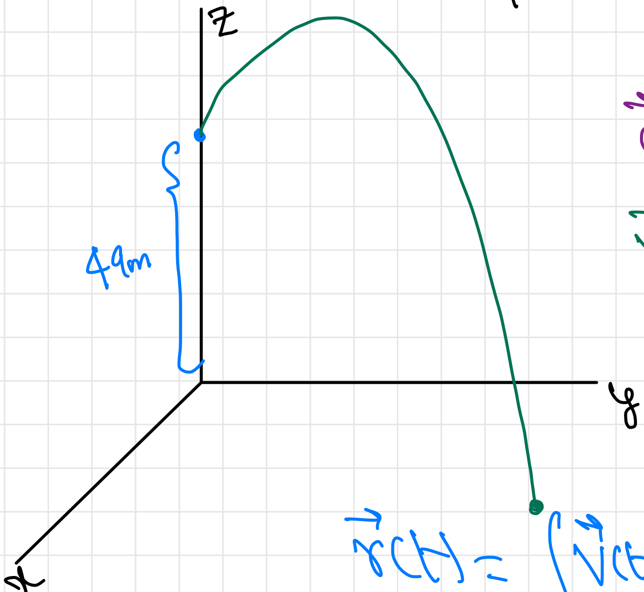
$$\int_0^1 \vec{r}(t)\,dt = \left\langle \frac{t^3}{3}, e^t, \frac{2}{3}t^{3/2} \right\rangle \Big|_{t=0}^{t=1}$$

$$= \left\langle \frac{1}{3}, e, \frac{2}{3} \right\rangle - \langle 0, 1, 0 \rangle$$

$$= \left\langle \frac{1}{3}, e-1, \frac{2}{3} \right\rangle$$

eg: A ball 49m above the ground is launched with initial velocity of $\vec{v}(0) = \langle 2, 4, 14.7 \rangle$ m/s. Find the position vector $\vec{r}(t)$.

initial velocity




$\vec{r}(0) = \langle 0, 0, 49 \rangle$ m

$\vec{a}(t) = \langle 0, 0, -9.8 \rangle$ m/s$^2$

$\vec{v}(t) = \int \vec{a}(t)\,dt + \vec{C} = \langle 0, 0, -9.8t \rangle + \vec{C}$

$\langle 2, 4, 14.7 \rangle = \vec{v}(0) = \vec{C}$

$\vec{v}(t) = \langle 2, 4, -9.8t + 14.7 \rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt + \vec{D} = \langle 2t, 4t, -4.9t^2 + 14.7t \rangle + \vec{D}$

$\langle 0, 0, 49 \rangle = \vec{r}(0) = \langle 0, 0, 0 \rangle + \vec{D}$

$\vec{r}(t) = \langle 2t, 4t, -4.9t^2 + 14.7t + 49 \rangle$.

Q: When does the ball hit the ground?

$z = 0$

$$\vec{r}(t) = \langle \underbrace{2t}_{x(t)}, \underbrace{4t}_{y(t)}, \underbrace{-4.9t^2 + 14.7t + 49}_{z(t)} \rangle$$

$$-4.9t^2 + 14.7t + 49 = 0$$

$$-4.9(t^2 - 3t - 10) = 0$$

$$-4.9\,(t-5)(t+2) = 0 \rightsquigarrow t = 5 \text{ or } t = -2$$

$$\vec{r}(5) = \langle 10, 20, 0 \rangle$$

Q. When does the ball attain its peak height?

$\underbrace{\text{peak height}}_{\text{velocity component in } z \text{ direction } = 0}$

$V(t) = \langle 2, 4, \underbrace{-9.8t + 14.7}_{=0} \rangle$

$-9.8t + 14.7 = 0 \rightarrow t = \frac{14.7}{9.8} = \frac{3}{2} = 1.5$ sec.

$\vec{r}(t) = \langle \underbrace{2t}_{x(t)}, \underbrace{4t}_{y(t)}, \underbrace{-4.9t^2 + 14.7t + 49}_{z(t)} \rangle$

Max. height $=$ z- component at $t = 1.5$

$= -4.9(1.5)^2 + 14.7(1.5) + 49$ m.