

Lesson 9: Functions of Several Variables (15.1)

Today: *

- * Functions of two (or three) variables
- * Domain
- * Sketching graphs

Next: *

- * Limits and Continuity

Office Hours: Monday, Friday 9:45 PM - 11:00 PM } MATH
Thursday 11:00 AM - 12:00 PM } 842
Thursday 12:15 PM - 1:15 PM (Windsor)

Warmup: Review functions of 1-Variable

① What is the Domain of $f(x) = \sqrt{9-x^2}$

all possible inputs

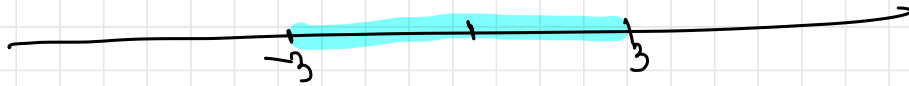
↑ input

output

$$9 - x^2 \geq 0$$

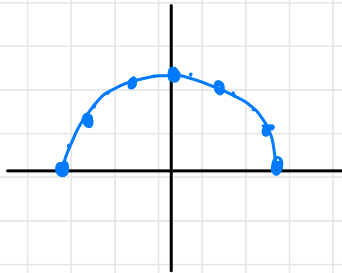
$$x^2 \leq 9$$

$$-3 \leq x \leq 3$$



② Sketch the graph of $f(x) = \sqrt{9-x^2} \Rightarrow y = \sqrt{9-x^2} > 0$

$(x, f(x))$ in $\mathbb{R}^2$



$$f(3) = 0$$

$$f(-3) = 0$$

$$f(2) = \sqrt{5}$$

$$f(-2) = \sqrt{5}$$

$$f(1) = \sqrt{8}$$

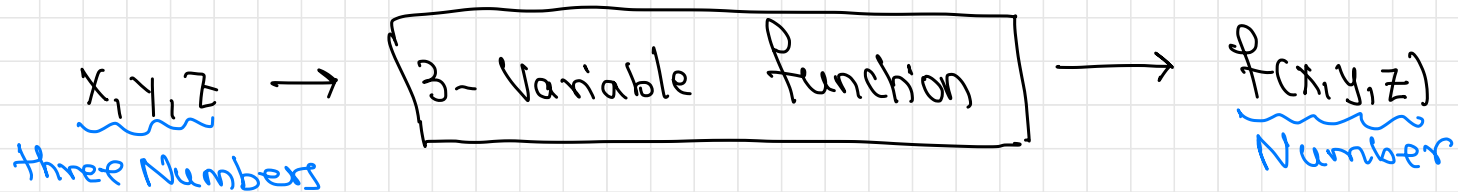
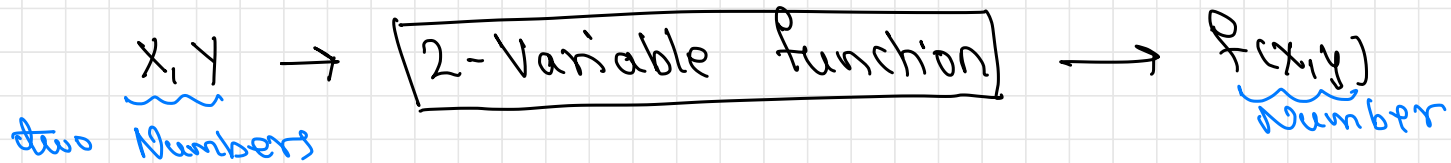
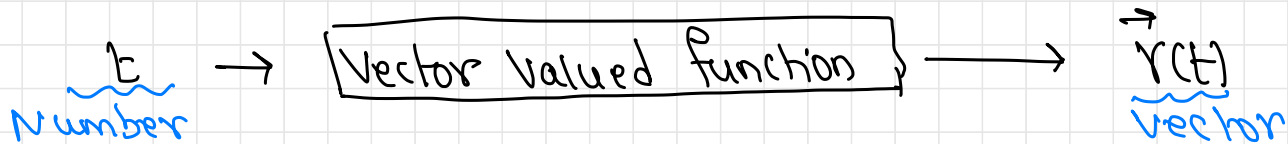
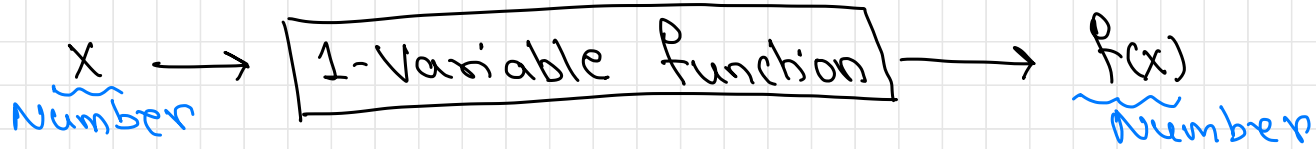
$$f(-1) = \sqrt{8}$$

$$f(0) = 3$$

$$y^2 = 9 - x^2$$

$$\left. \begin{array}{l} x^2 + y^2 = 9 \\ y > 0 \end{array} \right\} \text{Semi Circle}$$

Functions of two (or more) Variables



Examples: ① $f(x,y) = x^2 + y^2$

$$f(1,3) = 1 + 9 = 10$$

$$f(-10,11) = 100 + 121 = 221$$

$$\textcircled{2} f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$$

$$f(-10,13) = \frac{\sqrt{-10+13+1}}{-10-1} = \frac{2}{-11}$$

$$f(1, \underline{5}) = \text{X}$$

$$\textcircled{3} f(x,y,z) = \frac{\sqrt{9-x^2-y^2}}{\sqrt{\underline{25-z^2}}}$$

$$f(0,0,0) = \frac{3}{5}$$
$$f(0,1,3) = \frac{\sqrt{8}}{\sqrt{16}}$$

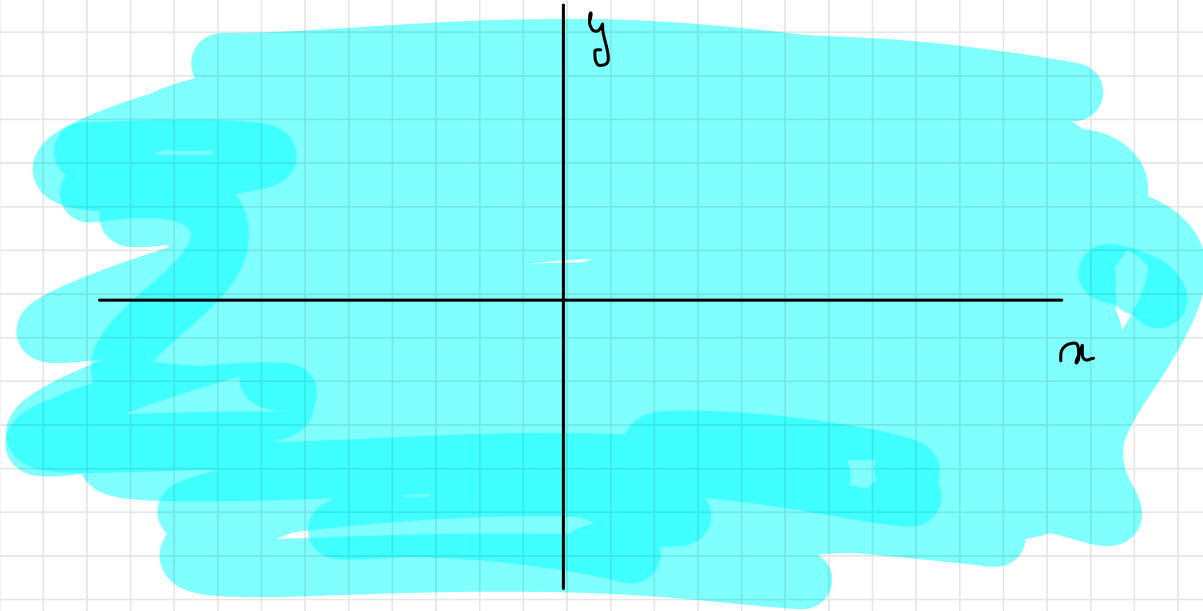
$$f(0,1, \underline{5}) = \text{X}$$

CANNOT have $z=5$ in input

Domain } all possible inputs
Region in $\mathbb{R}^2$ for function of 2-Variables
Region in $\mathbb{R}^3$ for function of 3-Variables

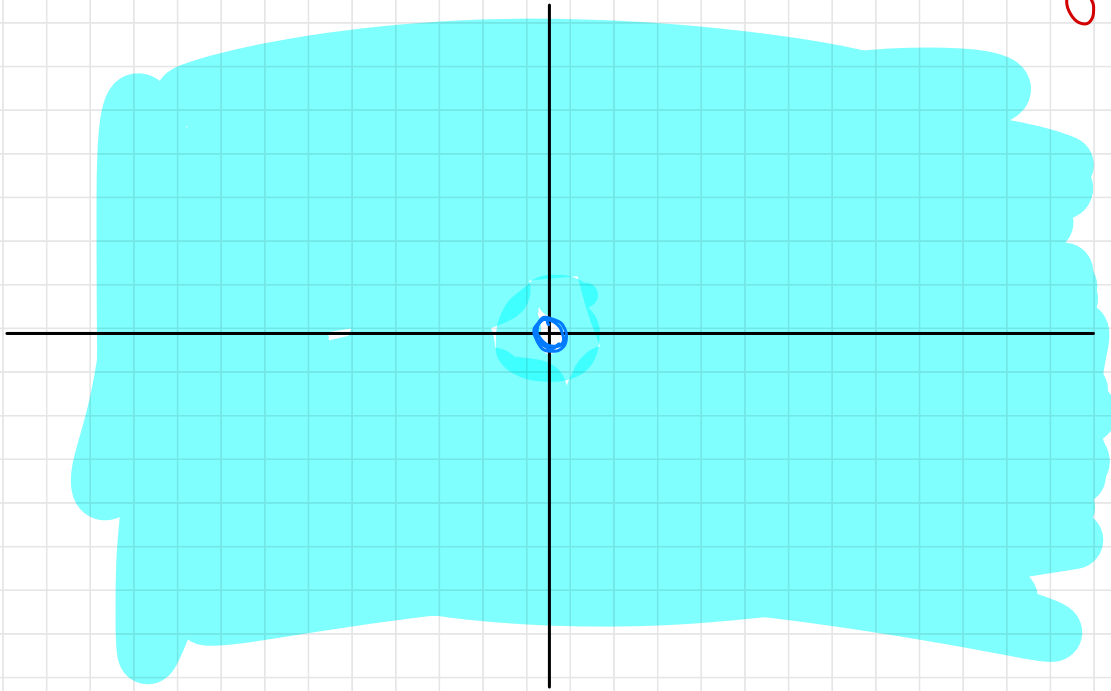
eg. $f(x, y) = x^2 + y^2$

Domain is all of $\mathbb{R}^2$



eg! $f(x,y) = \frac{1}{x^2+y^2}$

Denominator = $x^2+y^2 \neq 0$
(0,0) is not
in Domain



○ $\rightarrow$ point
not
included

● $\rightarrow$ point
included

eg:

$$f(x,y) = \sqrt{\frac{x+y+1}{x-1}} \geq 0$$

$x \neq 1 \Rightarrow x=1$ is not in Domain

Dotted line to represent it's not included

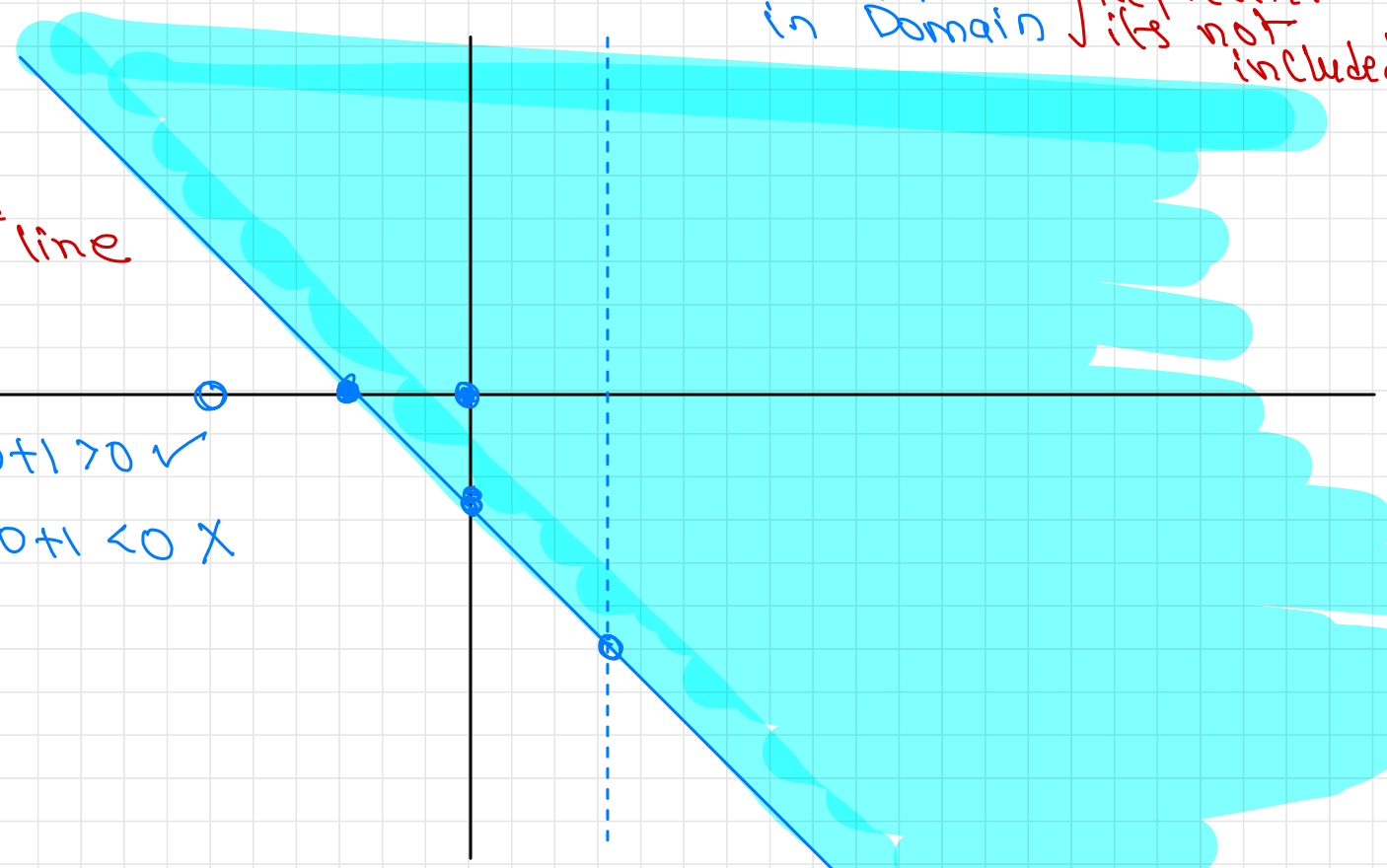
$$x+y+1 \geq 0$$

$$x+y+1 = 0$$

included
Represented
by solid line

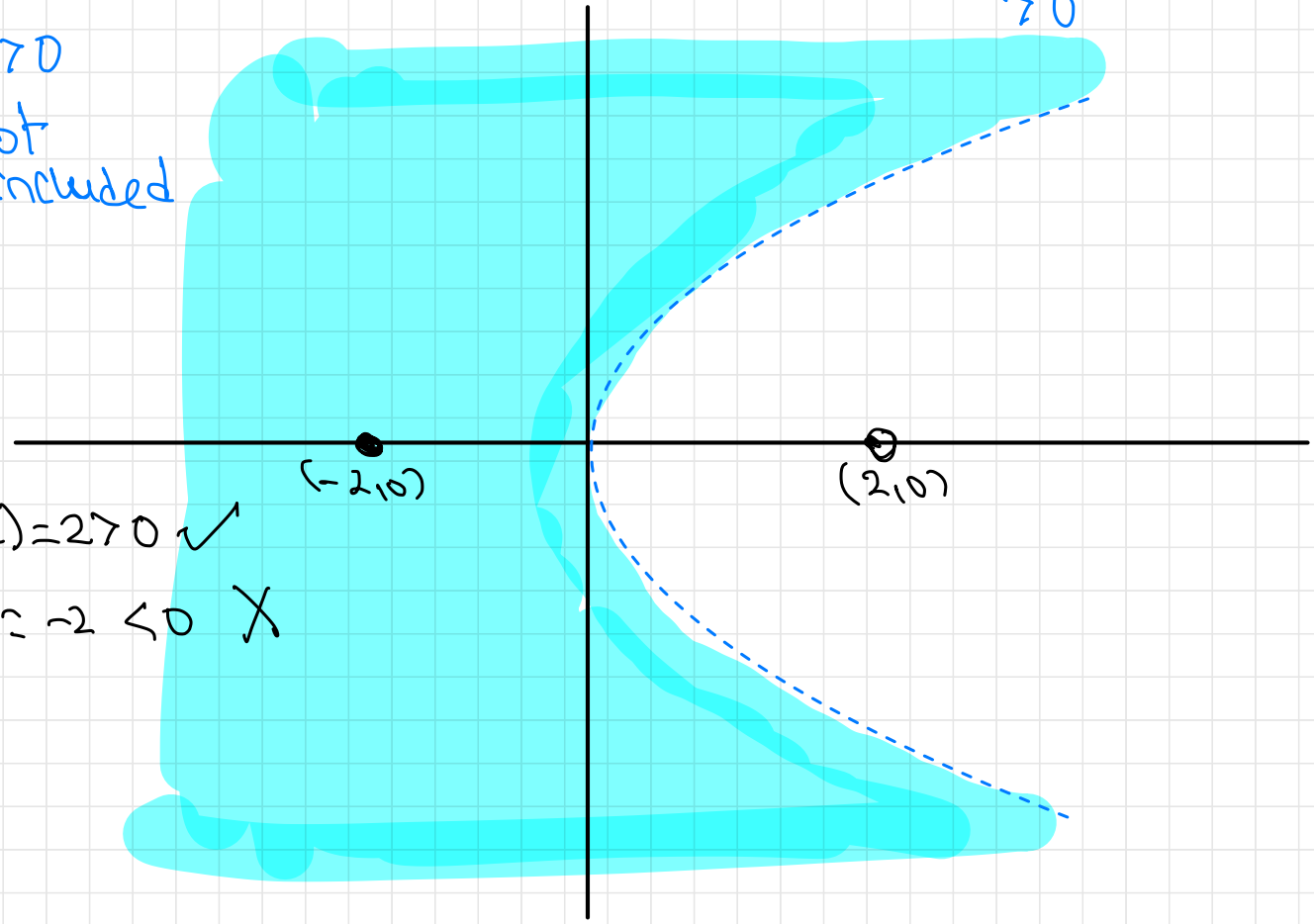
$$(0,0) \Rightarrow 0+0+1 \geq 0 \checkmark$$

$$(-2,0) \Rightarrow -2+0+1 < 0 \times$$



eg: Find the Domain of $f(x,y) = y \ln(y^2 - x)$

$y^2 - x > 0$
 $y^2 = x$ not included



$0^2 - (-2) = 2 > 0 \checkmark$
 $0^2 - (2) = -2 < 0 \times$

Break!!

Stretch

Walk around

Ask Questions

Reflect

Talk to your neighbors

- - - - -

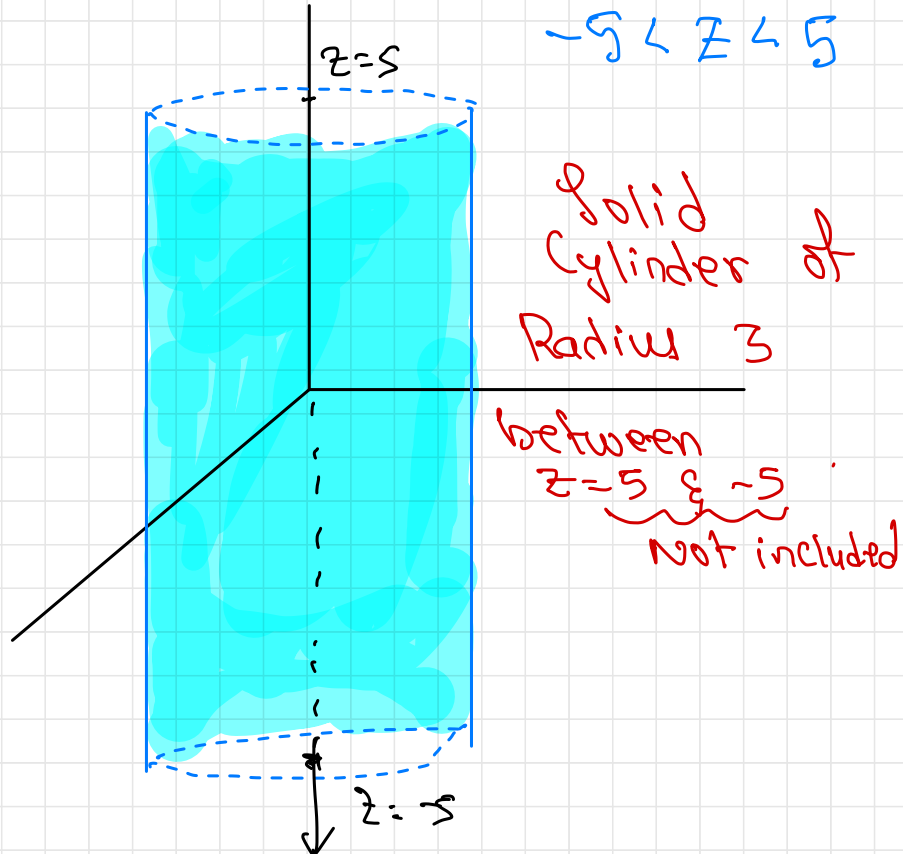
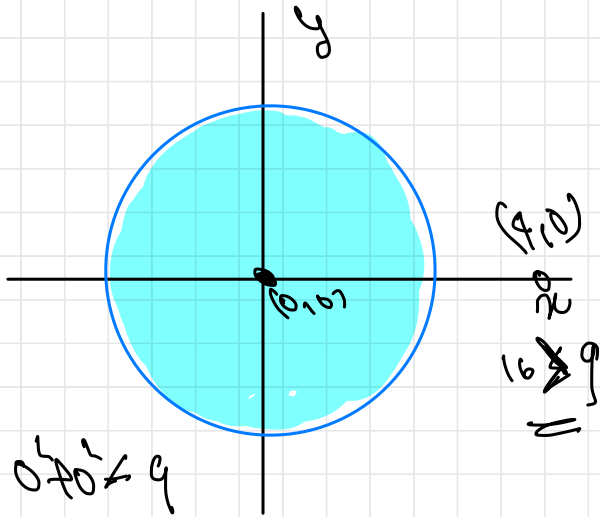
Next: * Domain of function of 3 variables

* Graphs

eg: Find the domain of $f(x,y,z) = \frac{\sqrt{9-x^2-y^2}}{\sqrt{25-z^2}}$

$9-x^2-y^2 \geq 0$
 $x^2+y^2 \leq 9$

$25-z^2 > 0$
 $-5 < z < 5$



Graph of $f(x,y) \rightsquigarrow (x,y, f(x,y))$ in $\mathbb{R}^3$
for all (x,y) is Domain.

Sketch: $\downarrow$ $z = f(x,y)$ Surface

By taking cross sections

OR
Traces

OR
Level curves

OR
Contours

for different possible
values of z .

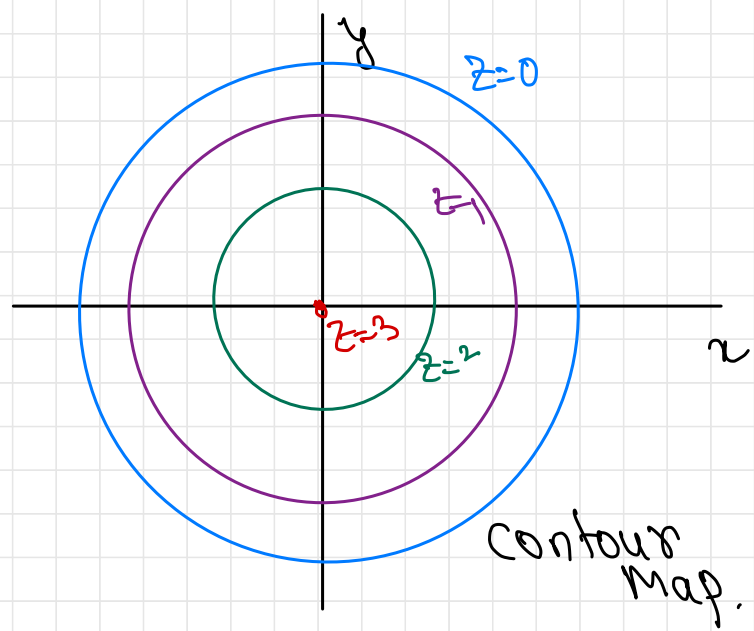
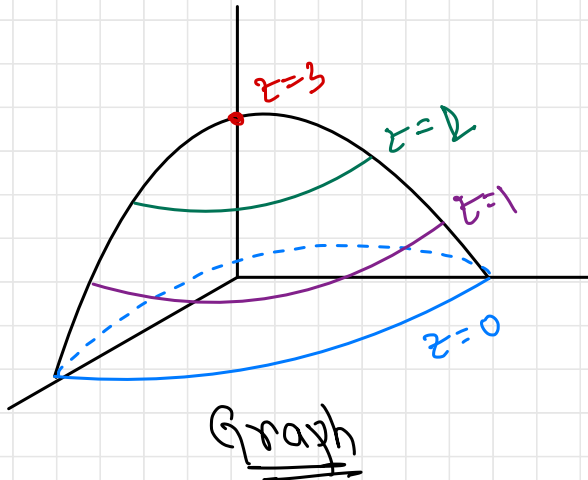
eg: $f(x,y) = \sqrt{9-x^2-y^2}$
 $z = \sqrt{9-x^2-y^2}$

$z=0 \rightarrow \sqrt{9-x^2-y^2}=0 \Rightarrow x^2+y^2=9$

$z=1 \rightarrow \sqrt{9-x^2-y^2}=1 \rightarrow x^2+y^2=8$

$z=2 \rightarrow x^2+y^2=5$

$z=3 \rightarrow x^2+y^2=0$



$z = \sqrt{9-x^2-y^2} > 0$

$z^2 = 9-x^2-y^2$

$x^2+y^2+z^2=9$

$z > 0$

$\frac{1}{2}$ of sphere
 Radius 3.

#1: $f(x, y) = x^2 + y^2$

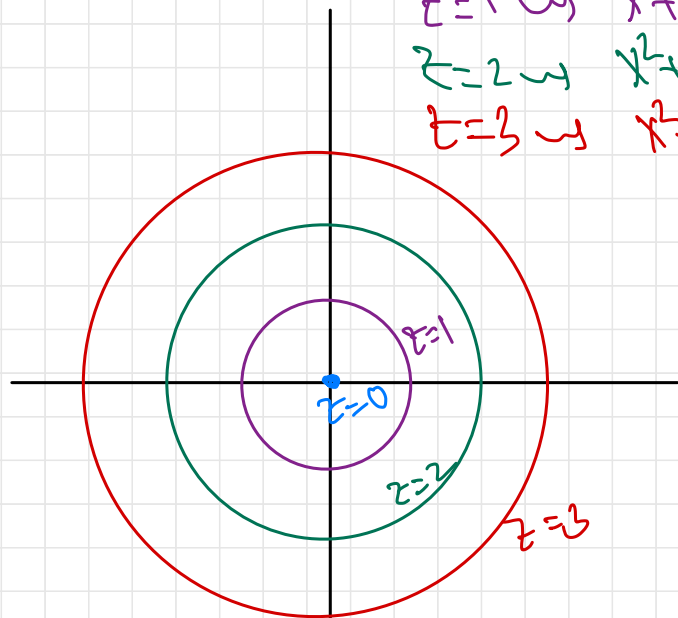
$$z = x^2 + y^2$$

$$z=0 \leadsto x^2 + y^2 = 0$$

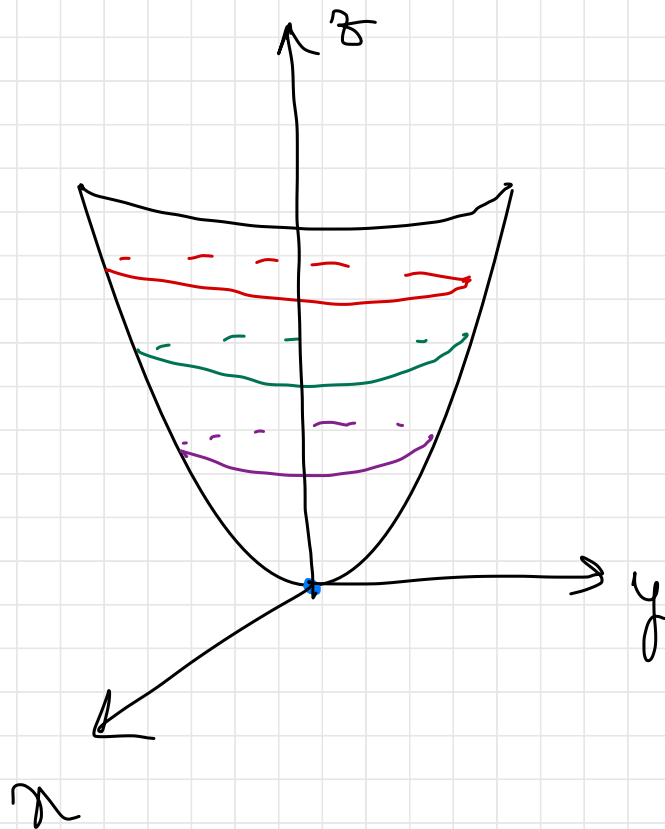
$$z=1 \leadsto x^2 + y^2 = 1$$

$$z=2 \leadsto x^2 + y^2 = 2$$

$$z=3 \leadsto x^2 + y^2 = 3$$



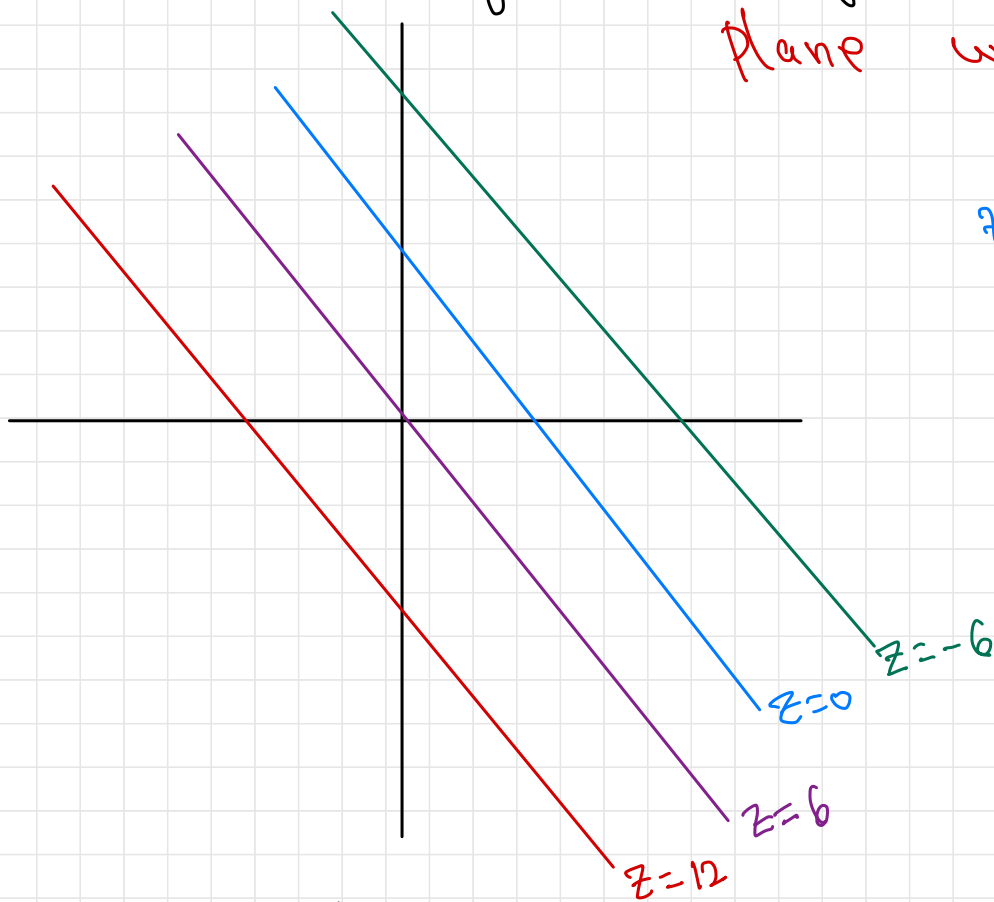
$z < 0$



e.g.: $f(x,y) = 6 - 3x - 2y$

$$z = 6 - 3x - 2y \rightarrow 3x + 2y + z = 6$$

plane with Normal vector
 $\langle 3, 2, 1 \rangle$



$$z=0 \rightarrow 3x + 2y = 6$$

$$z=6 \rightarrow 3x + 2y = 0$$

$$z=-6 \rightarrow 3x + 2y = 12$$

$$z=12 \rightarrow 3x + 2y = -6$$