Method of Elimination

Ex: $\begin{cases} x' = 4x + y + 2t \\ y' = -2x + y \end{cases}$ (2)

$D = \frac{d}{dt}$

$\begin{cases} Dx = 4x + y + 2t \\ Dy = -2x + y \end{cases}$

L_1 L_2 $f_1(t)$

$(D-4)x - 1\cdot y = 2t$ | $L_1 x + L_2 y = f_1(t)$

L_3 L_4 $f_2(t)$

$2\cdot x + (D-1)y = 0$ | $L_3 x + L_4 y = f_2(t)$

$2(D-4)x - 2y = 4t$ | $L_3 L_1 x + L_3 L_2 y = L_3 f_1$

$(D-4)\cdot 2x + (D-4)(D-1)y = 0$ (-) | $L_1 L_3 x + L_1 L_4 y = L_1 f_2$ (-)

↳ works bec. $L_3 L_1 x = L_1 L_3 x$

$-2y - (D-4)(D-1)y = 4t$

$(L_3 L_2 - L_1 L_4)y = L_3 f_1 - L_1 f_2$

a diff'l operator

$(-D^2 + 5D - 6)y = 4t$

↳ undet. coef.

$-y'' + 5y' - 6y = 4t$

Char. eq: $r^2 - 5r + 6 = 0 \Rightarrow r = 2,\ r = 3$

$y_p = A + Bt$

$-y_p'' + 5y_p' - 6y_p = 0 + 5B - 6A - 6Bt = 4t$

$\Rightarrow B = -\frac{2}{3},\ A = -\frac{5}{9}$

$y = c_1 e^{2t} + c_2 e^{3t} - \frac{5}{9} - \frac{2}{3}t.$ (1)

To find x:

(2) $\Rightarrow x = -\frac{y' + y}{2}$

$$\Rightarrow x = -\frac{1}{2}\left(c_1 \cdot 2e^{2t} + c_2 \cdot 3e^{3t} - \frac{2}{3} + c_1 e^{2t} + c_2 e^{3t} - \frac{5}{9} - \frac{2}{3}t \right)$$

(2)

(1), (2) → soln to system. 2 free parameters.