Plan for Today:

1. Finish § 1.1
2. Start § 1.2

What to know from § 1.2:

1. What is a general solution and what is a particular solution
2. Be able to solve differential equations of the form $y' = f(x)$
3. Be able to solve problems involving linear motion, possibly vertical, by solving such differential equations.
4. Know the various physical units in FPS and MKS systems and be able to convert between them - in particular it would be good to memorize the value of the gravitational acceleration in FPS and MKS (it has appeared in finals in the past).

Reminders:

1. Read the textbook!
2. Fill out your availability for office hours here: https://whenisgood.net/3kzxd2i. You can do it anonymously if you so prefer.
3. Get a head start on the first bundle of homework assignments, due on Tuesday of week 3.
4. Have a nice weekend and stay safe.

Last time: diff'l eq'n.

Mentioned: Model physical phenomena using dif'l eq's.

Ex: Time rate of change of the # of people infected w/ a disease is proportional to the product of # of people who have it & # of people who don't. Assume P_0 is the total population.

$\{ N(t) \rightarrow$ # people who have disease.

$P_0 - N(t) \to$ # people who don't.

$N'(t) \to$ time rate of change.

$$\boxed{N'(t) = k\,(P_0 - N(t))\,N(t)} \quad (*)$$

dif'l eqn. (logistic dif. eqn)

k: constant of proportionality

<u>Claim:</u> $N(t) = \dfrac{P_0}{1+e^{-P_0(kt+c)}}$ solves $(*)$ for any C.

! When asked to check if a <u>given</u> function satisfies an ODE we don't have to solve the ODE!

Take dev. of $N(t)$, check $(*)$ is satisfied

$$N'(t) = -P_0 \frac{1}{\left(1+e^{-P_0(kt+c)}\right)^2}(-P_0 k)e^{-P_0(kt+c)}$$

$$(P_0 - N(t)) = \frac{P_0\left(1+e^{-P_0(kt+c)}\right) - P_0}{1+e^{-P_0(kt+C)}}$$

$$= \frac{P_0 e^{-P_0(kt+c)}}{1+e^{-P_0(kt+c)}}$$

$$N(t)(P_0 - N(t)) = \frac{P_0^2 \, e^{-P_0(kt+c)}}{\left(1 + e^{-P_0(kt+c)}\right)^2}$$

$$\Rightarrow \quad N'(t) = k\, N(t)(P_0 - N(t))$$

$N(t)$ <u>sol'n</u>.

k, P_0, C depend on information given by the model.

Tippecanoe covid cases

$$\begin{cases} \text{Oct } 1 \to 1{,}000 \\ \text{Oct } 30 \to 3{,}000 \\ P_0 = 100{,}000 \end{cases}$$

Use to determine k, C, P_0

$t \to$ time after Sept 30th

Use thousands as units.

$$\begin{cases} 1 = \dfrac{100}{1 + e^{-100(k \cdot 1 + c)}} \\[2ex] 3 = \dfrac{100}{1 + e^{-100(k \cdot 30 + c)}} \end{cases} \qquad \begin{aligned} k &\sim 4 \cdot 10^{-4} \\ c &\sim -0.05 \end{aligned}$$

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<u>Some more terminology.</u>

Order of a dif'l eq'n $\to$ highest order of derivative which appears.

Ex: $y'' + y = x^5 \rightarrow$ order 2.

$y^{(4)} + 3y^5 + y' = 3\sin(x)$ order 4.

derivative power

A sol'n of a dif. eq'n on an interval is a continuous function which satisfies dif. eq'n.

Ex: $y' = e^{-y}$

$y = \ln(x+c)$ (*)

$y' = \frac{1}{x+c}$, $e^{-y} = e^{-\ln(x+c)} = \frac{1}{x+c}$

(*) is a solution on the interval $x+c>0$ (otherwise $\ln(x+c)$ is not defined)

A family of solutions depending on a parameter c is called a general sol'n. (may or may not be the case that a general solution can produce every solution)

Ex: $y = \ln(x+c)$ is a general sol'n for $y' = e^{-y}$.

We can fix a c to obtain a particular solution.

Ex: for $C=1$, $y=\ln(x+1)$ is a part. sol'n to $y'=e^{-y}$

Common way to find C is when an initial condition is given by the problem: the value of a solution at a certain value of the independent variable.

Eg: want the solution of $y'=e^{-y}$ w/ $y(0)=0$. [in general such a sol'n may or may not exist]

$y=\ln(x+C)$ is a gen. sol'n.

Try to find a C that works.

$0=\ln(0+C) \rightarrow C=1$

$y=\ln(x+1)$ is a particular sol'n w/ $y(0)=0$.

1st order dif'l eq'n + initial condition = Initial value problem (IVP)

$$\left.\begin{cases} y'=f(x,y) \\ y(x_0)=y_0 \end{cases}\right\} \text{IVP}$$

Intro to § 1.2

In general a 1st order ODE is of form $F(x, y, y') = 0$

Ex: $y + (y')^2 = \sin(x)$

can be written as $F(x,y,y') = 0$

for $F(z,w,t) = w + t^2 - \sin(z)$

We'll look at cases where

$y' = f(x,y)$ (can solve for y', simpler).

In 1.2 even simpler case:

$$y' = f(x) \quad (*)$$

no dependence on y on RHS

E.g.: $y' = \sin(x)$

$y' = \dfrac{1}{\sqrt{1+x^2}}$

Not $y' = xy$

y on right hand side.

Can solve $(*)$ by integrating!

$y = \int f(x)\,dx + C$ general sol'n

Ex:

$y' = \sin(x) \Rightarrow y = \int \sin(x)\,dx \Rightarrow y = -\cos(x) + C$