Plan for today:
Finish § 1.2
Start § 1.3

Learning goals (in addition to the ones from last time)
1. Be able to roughly sketch the slope field corresponding to a first order ODE and solutions whose graphs pass through a given point.
2. Be able to use the slope field of an equation to predict its behavior.
3. Be able to use dfield to construct slope fields.

Reminders
1. Last chance to fill your availability for office hours. Please do it here: https://whenisgood.net/3kzxd2i, anonymously if you prefer.
2. Please enroll yourself to Piazza: https://piazza.com/class/kjzsmv75fxa11a
3. You can download the Java version of dfield here: https://www.cs.unm.edu/~joel/dfield/?_ga=2.88690011.1963541742.1610756889-1273638757.1609377145

Last time: $y' = f(x)$

only x dependence on RHS

y' isolated on LHS

$y' = \sin(x)$ ✓

$y' = y\sin(x)$ ✗

$y' = (y')^2 + 3\sin(x)$ ✗

Integrate:

$$y' = f(x) \Rightarrow \underbrace{y = \int f(x)\,dx + C}_{\text{general sol'n of } y' = f(x)}$$

Ex: $y' = \sin(x) \rightarrow y = \int \sin(x)\,dx = -\cos(x) + C$

Remarks:

1. Any 2 sol's of $y' = f(x)$ differ by a constant: they correspond to different values of C.
2. By fixing an initial condition we can

find a particular solution which satisfies it.

Ex: $y' = \sin(x)$

$y(\pi) = 3$

gen: $y = -\cos(x) + C$

$3 = y(\pi) = -\cos(\pi) + C \Rightarrow C = 2$

$\Rightarrow y = -\cos(x) + 2.$

3. Same for higher order of form $y^{(n)} = f(x)$:

$y'' = 0 \Rightarrow y' = C_1$

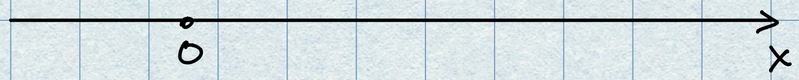
$\Rightarrow y = C_1 x + C_2$

2 free parameters.

(expect as many as the order of eqn)

Linear Motion

Object moving on straight line



choose origin on x axis.

$x = f(t) \rightarrow$ position at time t.

(if x axis points towards right then $f(t) > 0$ means we are on the right of the origin at time t)

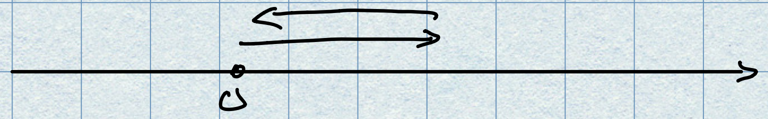
$v = f'(t) = \frac{dx}{dt}$ is velocity

! velocity is a signed quantity (positive or negative or 0) compare w/ <u>speed</u> $|v| \geq 0$

Given velocity: can recover position.

FTC: $$x(t) - \underline{\underline{x(t_0)}} = \int_{t_0}^{t} \frac{dx}{dt}(\tau)\,d\tau$$
$$= \int_{t_0}^{t} \underline{\underline{v(\tau)}}\,d\tau$$

! Position, not distance traveled!



Ex:

Start at origin. $v(t) = (1-t)(m/s)$, travel for 2 s.

$$x(t) - x(0) = \int_0^t (1-\tau)\,d\tau$$
$$= t - \frac{t^2}{2}$$

$\Rightarrow x(t) = t - \frac{t^2}{2}$

$x(2) = 0.$

To find distance traveled: need to know when velocity changes signs.

Concisely: distance traveled

$$\underline{\text{dis}(t)} = \int_0^t |v(\tau)|\,d\tau$$
$$= \int_0^t |1-\tau|\,d\tau \leftarrow \text{Exercise!}$$

Acceleration: $a(t) = \frac{dv}{dt}$

$$v(t) = v(t_0) + \int_{t_0}^{t} \frac{dv}{dt}(\tau)\, d\tau$$

$$= v(t_0) + \int_{t_0}^{t} a(\tau)\, d\tau \quad (1)$$

Important special case: $a(\tau) = \text{const} = a$
(object of constant mass moving under constant force)

$$F = m \cdot a$$

in this case $\quad v(t) = v(t_0) + a \cdot (t - t_0)$

Position: $x(t) = x(t_0) + \int_{t_0}^{t} v(\tau)\, d\tau \quad (2)$

if a const

$$= x(t_0) + \int_{t_0}^{t} v(t_0) + a(\tau - t_0)\, d\tau$$

$$\Rightarrow \boxed{x(t) = x(t_0) + v(t_0)(t - t_0) + a\frac{(t - t_0)^2}{2}}$$

Comment: Even if $a(t)$ not const. can plug in (1) into (2) to recover position.

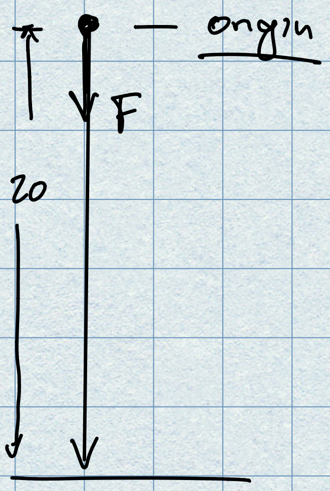
Ex: free fall w/ unknown gravitational const.

Let ball fall 20 ft from the ground

and it lands in 2s, in unknown planet. If we drop from 200 ft how long does it take?

More than one ways to set this up

1.



$v(0) = 0$ (let it fall)

F -> gravity w/ unknown gravitational accel. $\tilde{g}$.

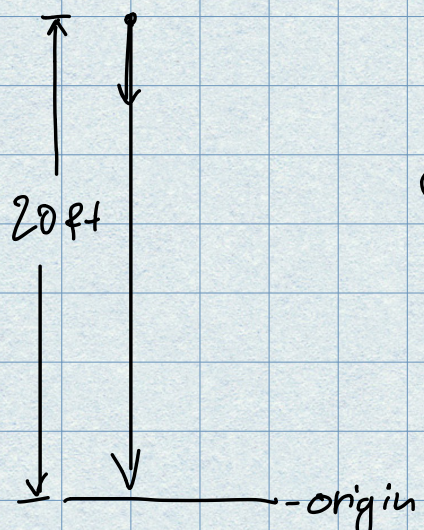
Position if origin is 20 ft from groun

$$y(t) = \underset{y(0)}{\overset{\shortparallel}{0}} + \underset{v(0)}{\overset{\shortparallel}{0}}(t-0) + \tilde{g}\frac{(t-0)^2}{2}$$

Know

$$y(2) = 20$$

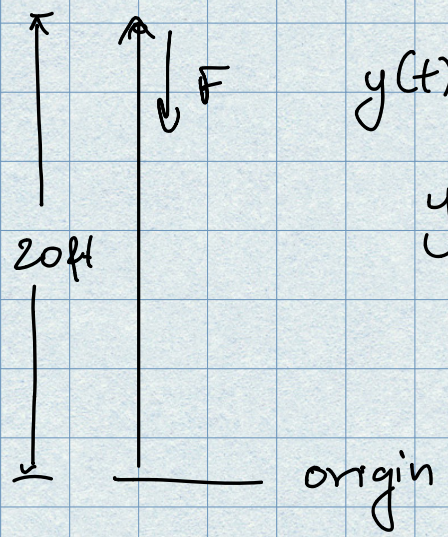
2.



$$y(t) = -20 + 0(t-0) + \tilde{g}\frac{(t-0}{2}$$

$$y(2) = 0$$

3.



$$y(t) = 20 + 0(t-0) - \tilde{g}\frac{(t-0)^2}{2}$$

$$y(2) = 0$$

Find $\tilde{g} = 10\,ft/s^2$

Knowing $\tilde{g}$, find how long it takes to get to the ground once you start 200 ft from ground.

(Exercise)