Plan for Today:

§ 1.3

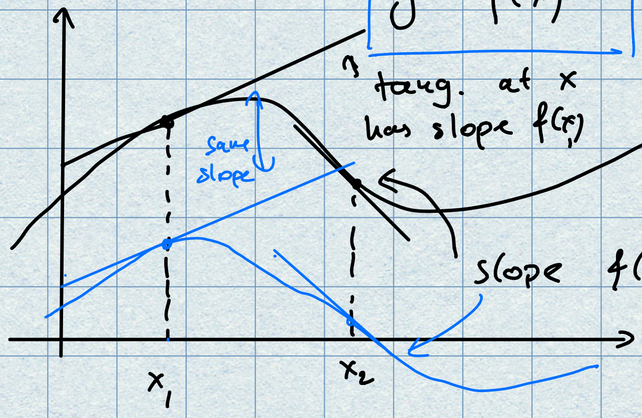
Learning goals

1. Be able to roughly sketch the slope field corresponding to a first order ODE and solutions whose graphs pass through a given point.
2. Be able to use the slope field of an equation to predict its behavior.
3. Be able to use dfield to construct slope fields.
4. Be able to decide whether the Theorem of Existence and Uniqueness of Solutions applies to a given IVP

Reminders

1. First HW set due next Tuesday. → On Gradescope & on MyLab Math
2. Office Hours Friday 2-3 pm
3. ODE Lounge every day 6.30-9.30 pm
4. Read the Textbook!

§ 1.3 In 1.2 saw how to solve $\boxed{y' = f(x)}$

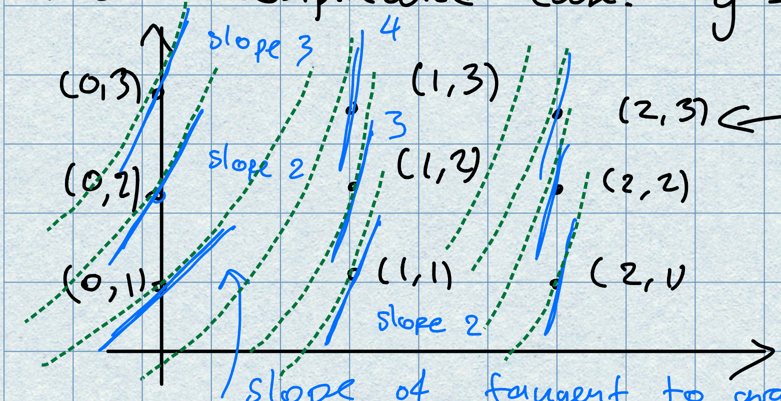


tang. at x has slope $f(x_1)$

graph of a sol'n $y = g(x)$ of $y' = f(x)$

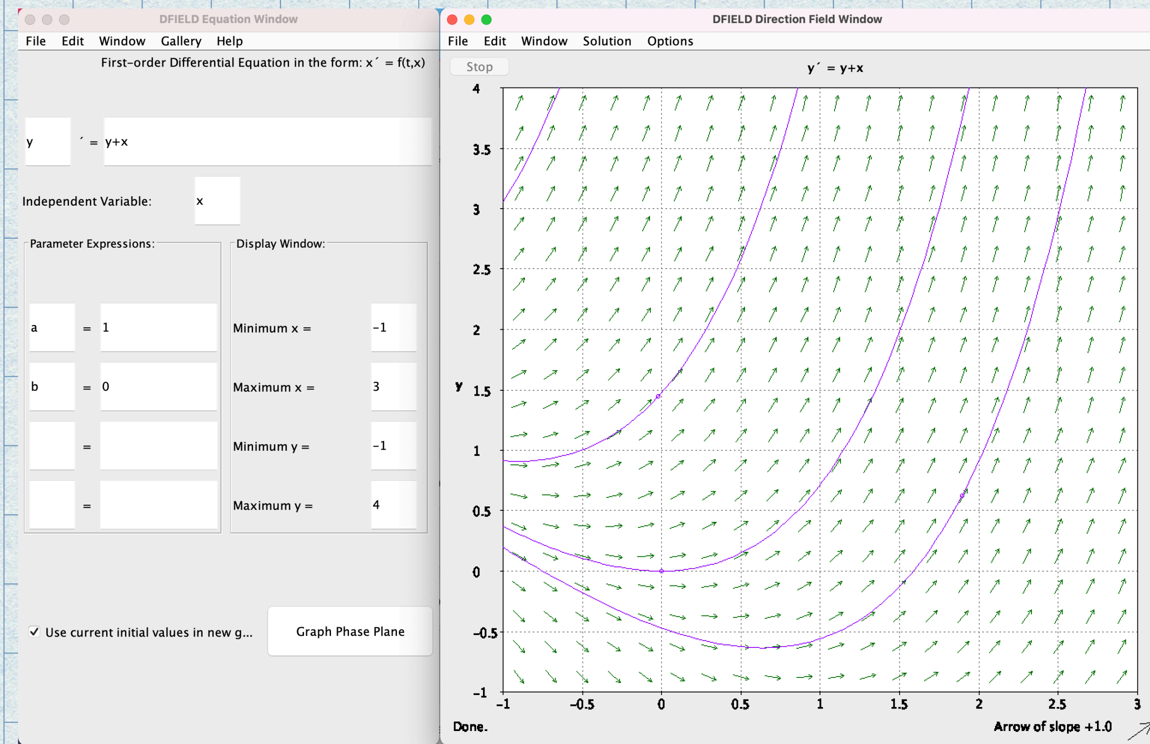
slope $f(x_2)$

More complicated case: $y' = y + x$



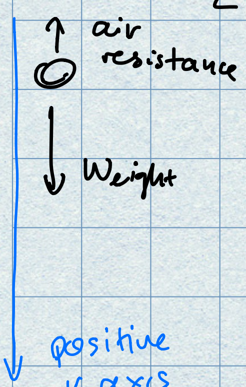
(2,3) ← $\boxed{\text{Slope field}}$

slope of tangent to graph of a sol. is $1+0=1$



Ex: Drop coin from a tall building

2 forces: gravity, air resistance.



gravity → gravitational acceleration

air resistance → deceleration proportional to velocity.

(know grav. accel. in ft/s² & m/s²)

Accel: $\frac{dv}{dt} = g - kv = 32 - 0.16v$

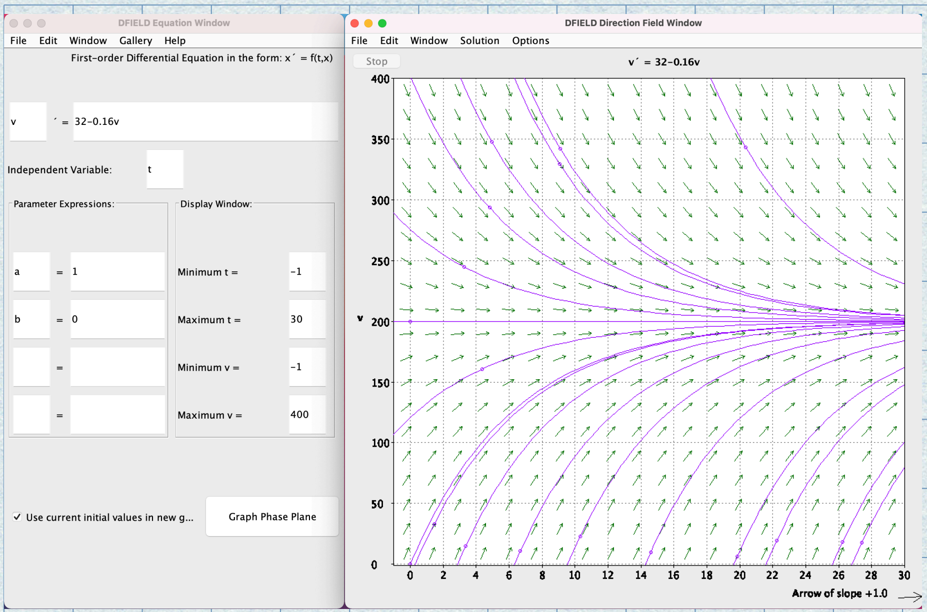
g: grav. accel.

k: a const.

0.16: experimental

$g = 32$ ft/s.

v: velocity.



See from dfield: asymptotically all sol's tend to 200 ft/s as $t \to \infty$

The constant $v = 200$ is a sol'n, called an <u>equilibrium sol.</u> //

<u>Existence & Uniqueness of Sol's</u>

Look at IVP (initial value problems).

Do we have sol's? How many?

i) $y' = \frac{1}{x}$

$\underbrace{y = \ln|x| + C}$ is a general sol'n.

can't solve $\begin{cases} y' = \frac{1}{x} \\ y(0) = 5 \end{cases}$: No sol'n is defined at 0, for any C.

ii) $\begin{cases} y' = \frac{4}{3} y^{\frac{1}{3}} \\ y(0) = 0 \end{cases}$

Note: $\left.\begin{array}{l} y = t^{4/3} \\ y = 0 \end{array}\right\}$ both solve IVP for $t \geq 0$

More than 1 sol's for IVP !

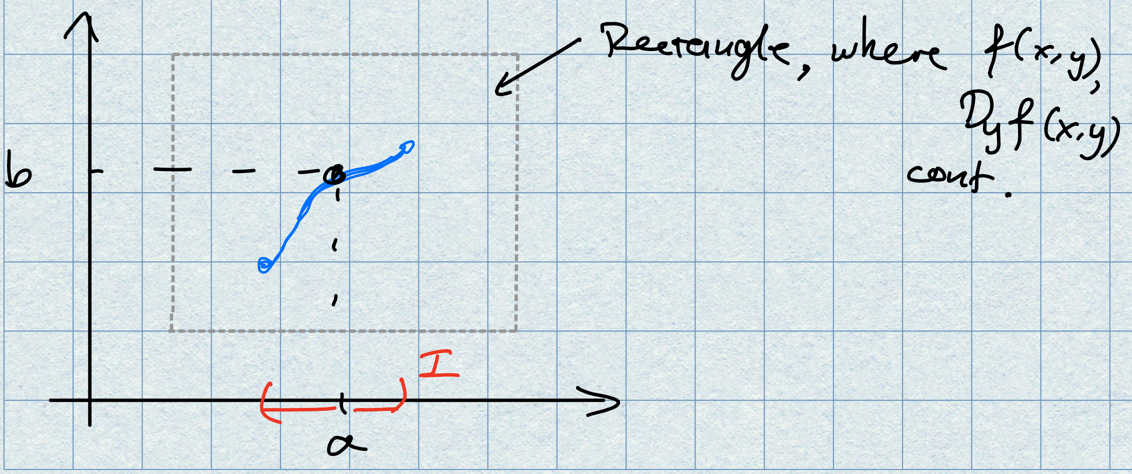
Existence & Uniqueness of Sols. Thm

S'pose: $f(x,y)$ & $D_y f = \frac{\partial f}{\partial y}(x,y)$ are continuous on a rectangle containing a pt (a,b) in its interior.

Then: There is an interval I containing a such that IVP

$$\begin{cases} \frac{dy}{dx} = f(x,y) \\ y(a) = b \end{cases}$$

has exactly one sol'n on I



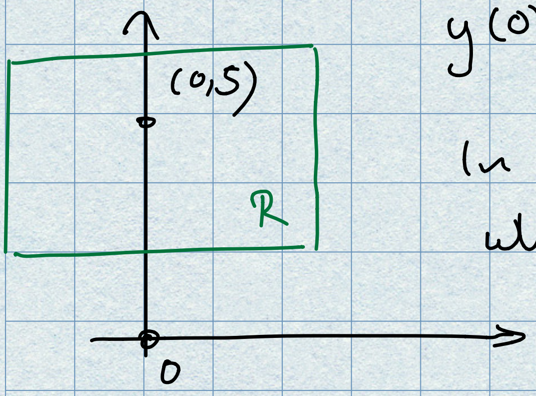
Note: I need not be as wide as the rectangle!

What happened in examples:

$y' = \frac{1}{x}$

$f(x,y) = \frac{1}{x}$

$y(0) = 5$



In any rectangle like R which contains $(0,5)$ $f(x,y)$ not continuous.

So: theorem does not apply.