Plan for Today:
Finish § 1.4

Learning goals for the day:
1. Be able to recognize and solve separable equations
2. Know what is an implicit, a general, and a singular solution of an ODE
3. Be able to solve problems involving population growth, radiocarbon dating and heating/cooling

Reminders:
1. First 3 HW assignments due tomorrow at 11.59 pm on both Gradescope (written) and MyLab Math (online)
2. Office hours: Today 1-2 pm, Tomorrow 8.30-9.30 am and 4-5 pm
3. Read the textbook, especially section on Natural Growth and Decay (you will need it for your homework).

Last time: separable ODE

$$\frac{dy}{dx} = k(y)g(x)$$

if $k(y) \neq 0$: $\frac{dy}{k(y)} = g(x)\,dx \Rightarrow \int \frac{dy}{k(y)} = \int g(x)\,dx$

$\Rightarrow H(y) = G(x) + C$

$H$ antider. for $\frac{1}{k}$

$G$ antider for $g$

Ex: $\frac{dy}{dx} = \frac{3-x}{y^2-y}$ (1)

Note: want $y^2 - y \neq 0$

(1) $\Rightarrow (y^2-y)\,dy = (3-x)\,dx$

$\Rightarrow \int y^2 - y\,dy = \int 3 - x\,dx$

$\Rightarrow \frac{y^3}{3} - \frac{y^2}{2} = 3x - \frac{x^2}{2} + C$

$\Rightarrow \underbrace{\frac{y^3}{3} - \frac{y^2}{2} - 3x + \frac{x^2}{2}}_{F(x,y)} = C$

Note: if $y(x)$ is a sol'n of ① then $F(x, y(x))$ is const.

Level set: for a function $F(x,y)$ a level set is a set of the form $\{(x,y) : F(x,y) = \text{const.}\}$

Ex: $F(x,y) = x^2 + y^2$ level sets are:

$\{(x,y) : F(x,y) = c\}$ is
- circle if $c > 0$
- pt if $c = 0$
- empty if $c < 0$

Found: graph of a sol'n is contained in a level set of $F(x,y)$

Def'n: Given an ODE, an eq'n $F(x,y) = 0$ such that for some solution $y(x)$ of the ODE we have $F(x, y(x)) = 0$ is called an implicit solution

Ex: $\frac{y^3}{3} - \frac{y^2}{2} - 3x + \frac{x^2}{2} - C = 0$ is an implicit solution of $\frac{dy}{dx} = \frac{3-x}{y^2 - y}$ for any $C$, as long as $y^2 - y \neq 0$.

(actually a general sol'n in implicit form since it depends on a parameter $C$)

If initial conditions are given: $y(-1) = -1$ we can find $C$.

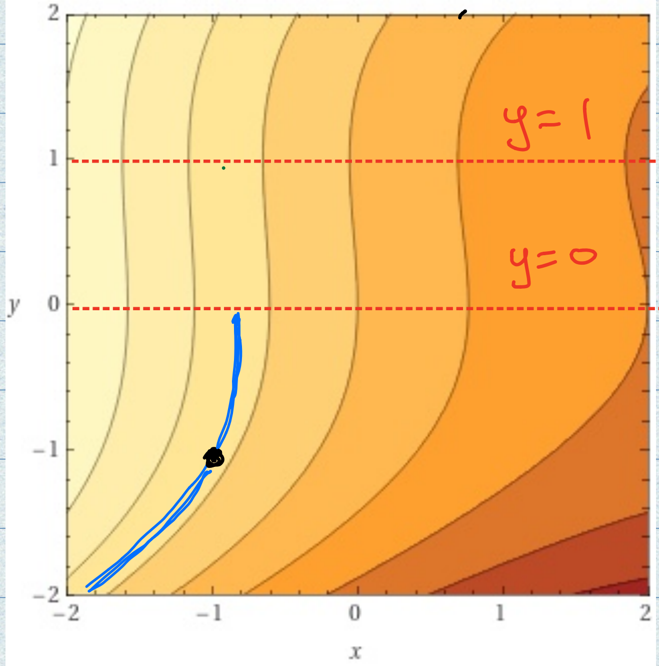
$$\frac{1}{3}(-1)^3 - \cancel{\frac{1}{2}(-1)^2} - 3(-1) + \cancel{\frac{1}{2}(-1)^2} - C = 0$$

$$\Rightarrow \quad C = 8/3$$

So: particular solution in implicit form:

$$\frac{y^3}{3} - \frac{y^2}{2} - 3x + \frac{x^2}{2} - \frac{8}{3} = 0,$$

as long as $y^2 - y \neq 0$, i.e. $y \neq 0$ & $y \neq 1$.




Graph of this sol'n: blue part in the picture, which avoids $y=0$

Ex: $\frac{dy}{dx} = y^2 x \quad \dots \Rightarrow \quad \frac{dy}{y^2} = x\,dx$ $\quad y \neq 0 \rightarrow$

$y \neq 0$

$$\int \frac{dy}{y^2} = \int x\,dx \Rightarrow -\frac{1}{y} = \frac{x^2}{2} + C$$

$$\underbrace{-\frac{1}{y} - \frac{x^2}{2} - C = 0}$$

implicit general sol'n.

$$\frac{1}{y} = -\frac{x^2}{2} - C$$

$$y = -\frac{1}{\frac{x^2}{2} + C} \Rightarrow \boxed{y = -\frac{2}{x^2 + 2C}}$$

(*) general sol'n

Note: $y(x) \equiv 0$ (const. fct) satisfies $\frac{dy}{dx} = y^2 x$

Can't obtain this sol'n for any value of $C$ in (*) !

A sol'n to an ODE which can't be obtained from a general sol'n is called a singular sol'n.

Ex: $y = 0$ is a singular sol'n of $\frac{dy}{dx} = y^2 x$

If all sol'ns of an ODE are given by a certain general sol'n (no singular sols exist) then this general sol'n is called the general sol'n.

Wednesday: Population growth.