Computer Project 1

Submission Instructions

1. Your submission should be a printout from Matlab containing the code, plots, responses and functions.

2. Please make sure that it is easy for the grader to see your response to each question: for each of the questions 1-3, please submit to Gradescope the **plots** and the **code** you used to generate them, followed by your **response** to the question. Please make sure that you match each answer to the corresponding designated question on Gradescope. For the functions, see Item 2:

3. For this assignment you will have to create Matlab functions (`*.m` files) that will be called by the code you write to answer the individual questions. Please include the functions separately in the designated Gradescope question named "Functions".

Resources

1. You will need access to Matlab. You can find instructions on how to obtain it here:

   `https://engineering.purdue.edu/ECN/Support/KB/Docs/MatlabToolboxes`

2. A short introductory video to Matlab and ode45 can be found in the Brightspace gallery.

   `https://purdue.brightspace.com/d2l/ext/rp/215961/lti/framedlaunch/8bdcb694-9ae4-45`

3. Documentation for `ode45` and `plot`:

   - `https://www.mathworks.com/help/matlab/ref/ode45.html`
   - `https://www.mathworks.com/help/matlab/ref/plot.html`

4. Also see the Matlab tutorials in the 266 Course Website, under "Resources":

   `https://www.math.purdue.edu/academic/courses/coursepage?subject=MA&course=26600`

Computer Project 1. Nonlinear Springs

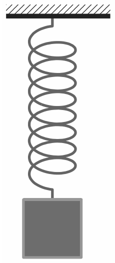
Goal: Investigate the behavior of nonlinear springs.

Tools needed: `ode45`, `plot`

Description: For certain (nonlinear) spring-mass systems, the spring force is not given by Hooke's Law but instead satisfies

$$F_{\text{spring}} = ku + \epsilon u^3,$$

where $k > 0$ is the spring constant and ϵ is small but may be positive or negative and represents the "strength" of the spring ($\epsilon = 0$ gives Hooke's Law). The spring is called a *hard spring* if $\epsilon > 0$ and a *soft spring* if $\epsilon < 0$.



Questions: Suppose a nonlinear spring-mass system satisfies the initial value problem

$$\begin{cases} u'' + u + \epsilon u^3 = 0 \\ u(0) = 0, \quad u'(0) = 1 \end{cases}$$

Use `ode45` and `plot` to answer the following:

1. Let $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$ and plot the solutions of the above initial value problem for $0 \le t \le 20$. Estimate the amplitude of the spring. Experiment with various $\epsilon > 0$. What appears to happen to the amplitude as $\epsilon \to \infty$? Let μ^+ denote the first time the mass reaches equilibrium after $t = 0$. Estimate μ^+ when $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$. What appears to happen to μ^+ as $\epsilon \to \infty$?

2. Let $\epsilon = -0.1, -0.2, -0.3, -0.4$ and plot the solutions of the above initial value problem for $0 \le t \le 20$. Estimate the amplitude of the spring. Experiment with various $\epsilon < 0$. What appears to happen to the amplitude as $\epsilon \to -\infty$? Let μ^- denote the first time the mass reaches equilibrium after $t = 0$. Estimate μ^- when $\epsilon = -0.1, -0.2, -0.3, -0.4$. What appears to happen to μ^- as $\epsilon \to -\infty$?

3. Given that a certain nonlinear hard spring satisfies the initial value problem

$$\begin{cases} u'' + \frac{1}{5}\, u' + \left(u + \frac{1}{5}\, u^3\right) = \cos \omega t \\ u(0) = 0, \quad u'(0) = 0 \end{cases}$$

plot the solution $u(t)$ over the interval $0 \le t \le 60$ for $\omega = 0.5, 0.7, 1.0, 1.3, 2.0$. Continue to experiment with various values of ω, where $0.5 \le \omega \le 2.0$, to find a value ω^* for which $|u(t)|$ is largest over the interval $40 \le t \le 60$.