Computer Project 2

Submission Instructions

1. Your submission should be a printout from Matlab containing the code, plots, responses and functions.

2. Please make sure that it is easy for the grader to see your response to each question: for each of the questions 1-2, please submit to Gradescope the **plots** and the **code** you used to generate them, followed by your **response** to the question. Please make sure that you match each answer to the corresponding designated question on Gradescope. For the functions, see Item 2:

3. For this assignment you will have to create Matlab functions (`*.m` files) that will be called by the code you write to answer the individual questions. Please include the functions separately in the designated Gradescope question named "Functions".

Resources

1. You will need access to Matlab. You can find instructions on how to obtain it here:

   `https://engineering.purdue.edu/ECN/Support/KB/Docs/MatlabToolboxes`

2. Documentation for `ode45` and `plot`:

   - `https://www.mathworks.com/help/matlab/ref/ode45.html`
   - `https://www.mathworks.com/help/matlab/ref/plot.html`

3. Also see the Matlab tutorials in the 266 Course Website, under "Resources":

   `https://www.math.purdue.edu/academic/courses/coursepage?subject=MA&course=26600`

4. Some context on RLC circuits and their relationship to spring mass systems can be found in the first 2 pages of Section 3.7 in the textbook. In short, the displacement $x(t)$ of a mass m attached to a spring with spring constant k in the presence of damping with damping constant c and external foce $F(t)$ is described by the differential equation

$$mx''(t) + cx'(t) + kx(t) = F(t). \tag{1}$$

This equation above has the same form as the one for the charge $Q(t)$ stored in the capacitor in an RLC circuit (equation $(*)$ below). This implies that the behavior of the spring-mass system can be studied, and phenomena such as presence or not of practical resonance can be predicted, by constructing an equivalent RLC circuit (which is easier/cheaper), studying the behavior of the charge, and interpreting the results in the context of the spring-mass system. Given a mass spring system with parameters as above, you can construct an equivalent RLC circuit by using a capacitor of capacitance $C = 1/k$, inductor of inductance $L = m$, resistor of resistance $R = c$ and voltage $E(t) = F(t)$.

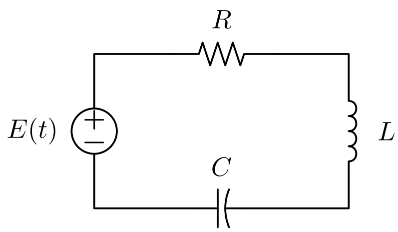
Computer Project 2. RLC Circuits

Goal: Investigate the charge on a capacitor in an RLC circuit with varying voltage.

Tools needed: `ode45`, `plot`

Description: If $Q(t)$ = charge on a capacitor at time t in an RLC circuit (with R, L and C being the resistance, inductance and capacitance, respectively) and $E(t)$ = applied voltage, then Kirchhoff's Laws give the following 2^{nd} order differential equation for $Q(t)$:

$$L\,Q''(t) + R\,Q'(t) + \frac{1}{C}\,Q(t) = E(t) \qquad (*)$$



Questions: Assume $L = 1$, $C = 1/5$, $R = 4$ and $E(t) = 10\cos\omega t$.

1. Use `ode45` (and plot routines) to plot the solution of $(*)$ with $Q(0) = 0$ and $Q'(0) = 0$ over the interval $0 \le t \le 80$ for $\omega = 0, 0.5, 1, 2, 4, 8, 16$.

2. Let $A(\omega)$ = maximum of $|Q(t)|$ over the interval $30 \le t \le 80$ (this approximates the amplitude of the steady-stat solution). Experiment with various values of ω and discuss what appears to happen to $A(\omega)$ as $\omega \to \infty$ and as $\omega \to 0$. Also, interpret your findings in terms of an equivalent spring-mass system.

Remark: There is an analogy between spring-mass system and RLC circuits given by:

Spring-mass system	**RLC circuit**
$m\,u'' + c\,u' + k\,u = F(t)$	$L\,Q'' + R\,Q' + \frac{1}{C}\,Q = E(t)$
$u =$ Displacement	$Q =$ Charge
$u' =$ Velocity	$Q' = I =$ Current
$m =$ Mass	$L =$ Inductance
$c =$ Damping constant	$R =$ Resistance
$k =$ Spring constant	$1/C = (\text{Capacitance})^{-1}$
$F(t) =$ External force	$E(t) =$ Voltage