

Computer Project 3

Submission Instructions

1. Your submission should contain screenshots of the plots you use to answer the questions.
2. Please make sure that it is easy for the grader to see your response to each question: for each of the questions 1-4, please submit to Gradescope the **screenshots of your plots** followed by your **response** to the question. Please make sure that you match each answer to the corresponding designated question on Gradescope.
3. Please use the “Keyboard Input of Initial Function” feature of `pplane8` to input precise initial conditions for generating the plots.
4. You do not need to include screenshots of the equations you entered in `pplane8` to generate the plots.

Resources

You can use either the `Matlab` or the `Java` version of `pplane8`. You can find them on the 266 Course Website:

<https://www.math.purdue.edu/academic/courses/coursepage?subject=MA&course=26600>

Computer Project 3. Predator-Prey Equations

Goal: Investigate the qualitative behavior of a *nonlinear* system of differential equations.

Tools needed: pplane8

Description: A farmer has ladybugs and aphids in her fields. The helpful ladybugs (predator) eat the destructive aphids (prey) who devour her crops.

$$\text{Let } \begin{cases} x(t) = \text{aphid population (in millions) at time } t, \\ y(t) = \text{ladybug population (in millions) at time } t. \end{cases}$$

The farmer knows that the growth rates of the aphid and ladybug populations are given respectively by

$$\begin{cases} \frac{dx}{dt} = x(1 - y), \\ \frac{dy}{dt} = y(x - 1). \end{cases}$$

Questions: Assume there are initially 800,000 aphids and 400,000 ladybugs in all that follows below.

1. Use pplane8 to plot the trajectory through (0.8, 0.4). As t increases, describe what happens to each population. Is the aphid population ever smaller than 300,000? Are the aphids ever eradicated? Does the ladybug population ever exceed 2 million?
2. A fellow farmer suggests that she use pesticide to kill the aphids. She is reluctant because it also kills the helpful ladybugs and she prefers to have some ladybugs remaining to eat other destructive insects. If she were to use a pesticide, the growth rates would then become

$$\begin{cases} \frac{dx}{dt} = x(1 - y) - sx, \\ \frac{dy}{dt} = y(x - 1) - sy. \end{cases} \quad (*)$$

where $s \geq 0$ is a measure of the “strength” of the pesticide – the larger the s , the stronger the pesticide. Currently there are only two commercially available strengths: $s = 0.5$ and $s = 0.75$. Plot the trajectories for the new system of equations (*) with these values of s . Will the aphids ever be totally eliminated?

3. If she knows her crops will survive if the aphid population never exceeds 2.6 million, which strength (if any) would you recommend she use: $s = 0.0$ (no pesticide), $s = 0.5$, $s = 0.75$?
4. By special permission, she could get a pesticide with the maximum strength of $s = 1.5$. Plot this trajectory. What happens to the ladybugs and aphids if she uses this pesticide?